Investigation of bound and resonant states of the $2N\bar{N}$ system with the aid of the Faddeev equations

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The Faddeev method is used to find the energies of the bound states and resonances in the $2N\bar{N}$ system in the region below the threshold of disintegration into three baryons. The pairs t-paired t-matrix was approximated with the aid of the unitary pole approximation. Bound states and a large number of resonances with quite small widths, including states of the cluster type, were obtained for a number of sets of conserved quantum numbers.

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As shown by Shapiro and co-workers,[1] states of quasinuclear type, with a rich energy spectrum, should appear in baryon systems that include antinucleons. The $2N\bar{N}$ system was considered in[2] by the method of multidimensional spherical functions (K harmonics). Unfortunately, the K-harmonics method is not suitable for the description of cluster states, which can play an important role because of the large depth of the $N\bar{N}$ potential. We have therefore undertaken in the present study a solution of this problem with the aid of the Faddeev equations. In the derivation of the equations we used the generalized Pauli principle.[3] The equations obtained in this case are generalization of the equations for a three-nucleon system (see, e.g.,[4]) via introduction of an additional (charge) degree of freedom.

Starting with the fact that the force between the baryons is short-range (the Coulomb forces are disregarded), upper bounds were set for the orbital angular momenta of the relative motion of the pair of particles (L) and the third particle ("spectator") relative to the mass center of the pair (l). This leads to a system of a finite number of two-dimensional integral equations for the partial waves. The following approximations were used: 1) $N\bar{N}$ interactions were taken into account only in those states (α) for which the theory of [1] predicts the existence of nuclearly bound states, of quasinuclear mesons $(N\bar{N})_\alpha$, as well as $N\bar{N}$ interactions in the $^3S_1$ and $^1S_0$ states. 2) The contribution of the tensor forces was neglected. 3) Imaginary values were taken for $l\,(\leqslant 1)$. 4) The unitary pole approximation (UPA) was used for the paired t_α matrix, so that the equations could be reduced to one-dimensional form.

The number of integral equations in the system depends on the set of conserved quantum numbers of the system $(2N\bar{N})I\,(J^P)$ (I is the isospin, J is the total angular momentum, and P is the parity). Their maximum number decreases, for example, from 50 to 7 if the restriction $l\leqslant 4$, is replaced by inclusion of components only with $l=0$, for which the realignment processes should be the most substantial. In connection with assumption 4), we note that the t_α matrix is contained in the kernel of the Faddeev equations at the pair energy $z=E-(3/4m)\mathbf{Q}^2$, where E is the internal energy of the $2N\bar{N}$ system, $\mathbf{Q}$ is the spectator momentum in the c.m.s., and m is the nucleon mass. The quantity z is negative at $E<0$, since $\mathbf{Q}^2\geqslant 0$, and can therefore be close to the pole for the bound state $(-\epsilon_\alpha)$, where the UPA is a rigorous approximation.

The energy eigenvalues of the system and the positions of the resonances $(E=-E_r-i\Gamma_r/2)$, are determined by the equation $\Delta(E)=0$, where $\Delta(E)$ is the determinant of the corresponding homogeneous system of algebraic equations obtained with the aid of Gaussian quadratures from the system of integral equations. $\Delta(E)$ is the analog of the Jost function for the three-baryon system. In the region of the resonances we have $2NN.\ -\epsilon_{\max}<E<0$, where $\epsilon_{\max}$ is the maximum binding energy of the quasi-nuclear meson from among those considered at the given $I(J^P)$, the kernels of the Faddeev equations have singularities due to the poles of the t_α matrices for the bound states. They are bypassed by shifting the indicated poles in the UPA amplitude downward into the complex plane $(-\epsilon_\alpha\rightarrow\epsilon_\alpha-i\gamma_\alpha/2)$. For the $N\bar{N}$-scattering amplitude this is a natural procedure since, as shown in[5], allowance for the annihilation does indeed lead to such a shift, with $\gamma_\alpha=\Gamma_\alpha^{(a)}(\Gamma_\alpha^{(a)}$ is the annihilation width). If in the considered state the $2N\bar{N}$ component corresponding to the S wave for $(N\bar{N})_\alpha$, for which $\Gamma_\alpha^{(a)}$ is relatively large, does not play a significant role (for example, for cluster states with $L\neq 0$), then we can put $\gamma_\alpha=\Gamma_\alpha^{(a)}$. In this case the quantity Γ_r describes approximately the total width of the decay with annihilation taken into account. The limit $\gamma_a \to 0^+$ corresponds to the pure nuclear problem. The calculations were made for states with quantum numbers $I(J^P)$ equal to 1/2 ($3/2^+$) and 1/2 ($5/2^+$). In these states there is no component with $L = l = 0$. For the states 1/2 ($3/2^+$) account

TABLE I.

a	1	2	3	4	5
$^{2T+1,2S+1}L_J$	$^{31}P_1$	$^{33}P_1$	$^{11}P_1$	$^{13}P_2$	$^{13}P_1$
ϵ_a	20	53	54	194	440
$\Gamma_a^{(a)}$	7.4	9.2	8.3	9.8	11.5

was taken of five components with $l = 0$ (see Table I, where T is the isospin, S is the spin, L is the orbital angular momentum, J is the total angular momentum for the given quasinuclear meson $(N\bar{N})_a$, and the energies are given in MeV). For states 1/2 ($5/2^+$) we took three components: $^{13}P_2$ (194 MeV, $l = 0$), $^{33}S_1$ (168 MeV, $l = 1$), and $^{33}P_2$ (0 MeV, $l = 0$). Bound states were obtained at energies 441* and 517 MeV for 1/2 ($3/2^+$) and 195* and 870 MeV for 1/2 ($5/2^+$) (the asterisks mark levels of the cluster type).

In the resonant region we calculated the function $1/|\Delta(E)|^2$ at $\gamma_a = \Gamma_a^{(a)}$ (the annihilation was not taken into account in the calculation of the bound-state energies, i.e., it was assumed that $\gamma_a = 0$). A rather rich structure was observed. The resonances can be identified with sufficiently clearly pronounced maxima of $1/|\Delta(E)|^2$ (E_r is the position of the maximum and Γ_r is its width at half-height).

The calculation results reduce to the following: 1) Bound states and resonances with rather small widths were obtained. 2) Some states are of the cluster type. 3) Near-threshold states exist [in the $2N\bar{N} \to (N\bar{N}) + N$ channel].

In conclusion, it must be noted that it is hardly meaningful to compare the obtained numbers with the experimental data until the validity of the approximations listed above is investigated. However, the fact that the calculations yielded sufficiently narrow states of the cluster type, for which the indicated approximations are more justified, points to the desirability of experimentally searching for such resonances in the "nucleon+4 or 5 pions" channel.

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