Vacancy relaxation phenomena in solid He^3

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(Submitted 4 July 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 5, 276–280 (5 September 1978)

Relaxation absorption of the energy of an RF magnetic field on ferromagnetically polarized regions (FPR) produced by vacancies in solid He^3 is investigated. The time of formation of the FPR in the absence of a magnetic field is estimated.

PACS numbers: 67.80.Jd

A vacancy in solid He^3 polarizes the surrounding nuclear spins and produces a ferromagnetically ordered region.[1] This paper deals with the possibility of observing such regions by RF-spectroscopy methods.

Let an He^3 crystal with vacancies be situated in a magnetic field. We consider the ferromagnetically polarized region (FPR) around the vacancy, with a magnetic moment directed along the field. At a temperature $T \gg T_N \sim 1$ mK (T_N is the Néel temperature) of a nuclear antiferromagnet, the nuclear spins can be regarded as noninteracting. The free energy of an FPR of radius R in a magnetic field H is given by (cf.[1]):

$$F = \epsilon_0 + \frac{\pi^2\hbar^2}{2MR^2} + \frac{4}{3}\pi R^3 n\left[Ts(H) - \mu H \frac{\exp\frac{\mu H}{T}}{\operatorname{ch}\frac{\mu H}{T}}\right], \tag{1}$$

where ϵ_0 is the vacancy-formation energy, M is the effective mass of the vacancy, n is the number of He^3 per unit volume, $s(H)$ is the entropy of the system per particle and depends on the external magnetic field, and μ is the magnetic moment of the He^3 nucleus.

Recognizing that

$$s(m) = -\left[\frac{m}{n}\ln\frac{m}{n} + \left(1 - \frac{m}{n}\right)\ln\left(1 - \frac{m}{n}\right)\right], \tag{2}$$

where $m = [n\exp(\mu H/T)]/2\cosh(\mu H/T)$ is the concentration of the particles with nuclear spin directed along the field, we can rewrite (1) in the form

$$F = \epsilon_0 + \frac{\pi^2\hbar^2}{2MR^2} + \frac{4}{3}\pi R^3 nT\left(\ln 2\operatorname{ch}\frac{\mu H}{T} - \frac{\mu H}{T}\right). \tag{3}$$

 0021-3640/78/170252-04$00.60

The equilibrium radius of the FPR in a magnetic field is determined from the relation $\partial F/\partial R = 0$, and is given by

$$R_0(H) = \left[\frac{\pi \hbar^2}{4nTM\left(\ln 2\,\mathrm{ch}\frac{\mu H}{T} - \frac{\mu H}{T}\right)} \right]^{1/5}. \tag{4}$$

We consider the case when $H = H_0 + H_1 \exp(i\omega t)$; $\mu H_1 \ll \mu H_0 \ll T$. At each instant of time, the FPR radius relaxes to its equilibrium value, whose change is described by formula (4). The relaxation is via diffusion of the nuclear spin; this diffusion takes place over the entire space outside the FPR. In contrast to Ref. 2, where the transport of the spins through the FPR determined the mobility of this region as a whole, in this case the spins inside the FPR do not participate in the diffusion. When the dimensions of the immobile FPR are altered by the diffusion, energy is absorbed from the oscillating magnetic field.

For a body in an external medium (and the FPR in He^3 can be regarded as such), the rate of dissipation at constant temperature in the volume is determined from the relation

$$\dot{F} = -T\dot{S}, \tag{5}$$

where $\dot{S} = n\int_\Omega (\partial s/\partial m)\dot{m}dV$ is the rate of change of the total entropy of the FPR. The integration is over the entire diffusion region.

Smallness of the amplitude of the induced oscillations of the nonequilibrium radius of the FPR correspond to a weak disequilibrium of the system. We can confine ourselves to the quadratic term in the expansion of $s(m)$ in a series near the maximum.

Then

$$\frac{\partial s}{\partial m} = -\frac{4}{n^2}(m - m_0), \tag{6}$$

where m_0 is the maximum point for $s(m)$.

In the noninteracting-spins approximation we have the continuity equation:

$$\dot{m} + \mathrm{div}\,\mathbf{j} = 0, \tag{7}$$

where $\mathbf{j} = -D\nabla m$ is the density of the diffusion flux, and D is the coefficient of self diffusion. Taking (6) and (7) into account, we obtain after integrating by parts

$$\dot{S} = \frac{4}{n} D \int_\Omega (\nabla m)^2 dV. \tag{8}$$

The spatial distribution of the concentration $m(r)$ is determined from the diffusion equation. Specifying the flux density $\mathbf{j}$ through the FPR boundary as the boundary condition, we get

$$m(r) = \frac{n}{2\mathrm{ch}\dfrac{\mu H}{T}}\left[\exp\frac{\mu H}{T} - \frac{R_0^2\dot{R}\exp\left(-\dfrac{\mu H}{T}\right)\exp ik(r-R_0)}{D(1-ikR_0)r}\right], \tag{9}$$

where $k = \sqrt{\omega/2D}\,(1+i)$.

The "equation of motion" (5) is linearized near the values $R = R_0(H_0)$, and $H = H_0$. With the aid of (3), (8) and (9) we get the solution

$$R = R_0(H_0) + \frac{D}{R_0(H_0)}\exp\frac{\mu H_0}{T}\mathrm{ch}\frac{\mu H_0}{T}\frac{\dfrac{R_0^2}{l^2}+\left(\dfrac{R_0}{l}+1\right)^2}{\dfrac{R_0}{l}+1}\frac{\exp i(\omega t+\delta)}{\sqrt{\omega^2+\gamma^2}}\frac{\mu H_1}{T}, \tag{10}$$

where $\tan\delta = \omega/\gamma$,

$$\gamma = -5D\left[\frac{\exp\dfrac{\mu H_0}{T}\,\mathrm{ch}\dfrac{\mu H_0}{T}}{R_0(H_0)}\right]^2\frac{\dfrac{R_0^2}{l^2}+\left(\dfrac{R_0}{l}+1\right)^2}{\dfrac{R^0}{l}+1}\left(\ln 2\mathrm{ch}\frac{\mu H_0}{T} - \frac{\mu H_0}{T}\right).$$

and $l = \sqrt{2D/\omega}$ is the thickness of the "skin layer."

The imaginary part $\chi''(\omega)$ of the susceptibility is determined from the relation

$$\chi''(\omega) = \frac{2P(\omega)}{\omega H_1^2}$$

where $P(\omega)$ is the time-averaged power absorbed per unit volume. With the aid of (5), (8), (9), and (10) we get

$$\chi''(\omega) = 4\pi NnR_0^3\frac{\mu^2}{T}f\frac{R_0}{l}, \tag{11}$$

where N is the number of vacancies per unit volume;

$$f(z) = \frac{z^2(z+1)[z^2+(z+1)^2]}{z^4(z+1)^2 + \dfrac{25}{4}\left(\exp\dfrac{\mu H_0}{T}\mathrm{ch}\dfrac{\mu H_0}{T}\right)^4\left(\ln 2\mathrm{ch}\dfrac{\mu H_0}{T} - \dfrac{\mu H_0}{T}\right)^2[z^2+(z+1)^2]^2}$$

For $D \sim 3 \times 10^{-8}$ cm^2/sec (Ref. 3) the value $l \sim R_0$ corresponds to a frequency $\omega \sim 6 \times 10^6$ Hz. In the quasistationary state $l \gg R_0$ and $\chi''(\omega) \sim \omega$; in the opposite case of high frequencies we have $l \ll R_0$ and $\chi''(\omega) \sim \omega^{-1/2}$. The validity of formulas (10) and (11) is restricted to the region $l \gtrsim a$ (a is the distance between atoms), corresponding to frequencies $\omega \lesssim 10^6$ Hz.

At a temperature $T \sim 10^{-2}$ K, vacancy concentration $x \sim 10^{-5}$, in a constant field $H_0 \lesssim 100$ G, and a frequency $\omega \sim 10^6$ Hz the imaginary part of the susceptibility is of the order of 10^{-7}.

In conclusion, we calculate the time during which the vacancy forms an FPR in the absence of a field. It is expressed by the integral

$$t = -\frac{4}{5}\frac{nTMR_0^7}{D\pi\hbar^2}\int_0^{R_0(0)}\frac{dR}{R - R_0(0)}$$

which diverges logarithmically at the upper limit. Cutting off the integral at $R - R_0(0) \sim a$, we obtain

$$t = \frac{4}{5} D \frac{R_0^2(0)}{(4\ln 2)^5} \ln\frac{R_0(0)}{a},$$

which amounts to 10^{-8} sec in order of magnitude.

I am sincerely grateful to A.F. Andreev for attentive guidance and valuable advice, and to V.I. Marchenko and A.E. Meierovich for useful discussions.

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