Muon distribution among the fission fragments in mesic atoms

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The probability of observing a muon on a light and on a heavy fragment after "prompt" fission of a nucleus by a muon is calculated.

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One of the possible results of the $2p$–$1s$ (or $3p$–$1s$) transition of muons in a heavy mesic atom may be the fission of the atomic nucleus (the so-called "prompt" fission)[1]. This raises the question: what is the probability of observing the muon on the heavy or the light fragment after the fission? The answer to this question may be connected with the interpretation of various experiments in nuclear physics of fission fragments and in the physics of mesic atoms. In addition, the problem is of interest from a general theoretical point of view, as an example of the application of methods developed in the physics of atomic collisions to phenomena accompanying the fission of atomic nuclei.

The results presented here alter qualitatively the initial estimates of the probability of observing a muon on the light or heavy fragment (assumed to be 10 and 90% respectively in the calculation of Ref. 2 and 50 and 50% in Ref. 3).

At the initial instant, the muon is on the $1s$ level of the combined atom (atomic number $Z = 92$). Since the fragments move apart slowly compared with the muon velocity, the muon remains predominantly in its lowest ($1s\sigma$) energy state. As the distance R between the fragment centers tends to infinity, this term goes over to the $1s$ level of the heavy fragment ($Z_h = 54$). The probability of muon transition to the nearest excited level ($2p\sigma$), which goes over as $R \to \infty$ into the $1s$ state of the light fragment ($Z_l \sim 38$), is in this case exponentially small and is determined in the adiabatic approximation by the Landau formula,[4] but the parameters that enter in it can be chosen in various manners.

Since the energy difference of the two states between which the muon transition takes place remains almost constant as $R \to \infty$, it follows that in this case we cannot use the Landau–Zener parametrization employed in Ref. 3 to calculate the sought probability, so that the result of Ref. 3 is too high by two orders of magnitude.

In our situation we must use the formula proposed initially by Demkov[5] to calculate the probability of charge exchange in atomic collisions, when the electron transfer takes place at relatively large $R \approx R_0$ in a narrow ΔR interval. The result of[5] is represented here in a somewhat more accurate form.

According to the Landau method of complex trajectories[4] the transition probability is ($\hbar = m_\mu = c = 1$)

$$W = \exp\left(-2\,\mathrm{Im}\int^{t_c} \Delta E_{1,2}\,dt\right),$$

where $\Delta E_{1,2}$ is the splitting terms, and the integration extends to the point t_c where the levels cross in the complex-time plane. In the two-level approximation, we parametrize the matrix elements of the Hamiltonian H_{ij} in the basis of the states of the separated atoms at large R in the following manner[5]:

$$|H_{11} - H_{22}| \approx \text{const} \equiv \kappa; \qquad H_{12} \approx \beta\exp(-\gamma t) \equiv \beta\exp(-\lambda R),$$

where λ is defined as the logarithmic derivative of the asymptotic exchange-interaction potential [see, e.g., formula (1.9) in Ref. 6], calculated in the transition region R_0. The latter corresponds to the point of term crossing and is determined from the equation

$$\kappa^2 + 4\beta^2\exp(-2\gamma t) = 0,$$

whose solution is $t_c = t_0 + i\tau_0$. The values of t_0 and τ_0 are obtained from the equations

$$\kappa = 2|H_{12}|; \qquad \tau_0 = \pi/2\gamma. \tag{1a,b}$$

For the probability we obtain ultimately:

$$W = \exp(-\pi\kappa/\lambda v) \quad \text{at} \quad R = R_0, \tag{2}$$

where v is the relative velocity of fragments with charges Z_l and Z_h in the transition region R_0, where this velocity is smaller by a factor two or three than at $R \to \infty$, thus substantially decreasing W.

A similar formula was obtained earlier[5] by one of us by solving the corresponding nonstationary problem, but the parameter λ was chosen equal to $v(2I)^{1/2}$ (where I is the smaller of the ionization potentials), and to find R_0 the equation used in[5] in lieu of (1a) was $\kappa = |H_{12}|$. The use of the method of Ref. 6 to find λ (in place of the method proposed in Ref. 5) decreases by several times the probability that the muon will be dragged by the light fragment (if $Z_l = 38$ and $Z_h = 54$), and the use of Eq. (1a) to obtain R_0 (in lieu of the method proposed in Ref. 5) approximately doubles this probability.

Of great importance in muon problems is allowance for the finite dimensions of the nucleus. An investigation of the muon binding energy in the field of a nucleus with finite dimensions, as calculated by us, decreases the value of κ by about one-half and this, according to formulas (1) and (2), leads to further increase of the aforementioned probability by 3–4 times. As a result, the probability of observing the muon on the light fragment is 0.14% ($Z_l = 38$), 1.6% ($Z_l = 40$), 9.4% ($Z_l = 42$). However, in view of the decrease of the yield of fragments with close values of Z, the most substantial contribution to the average muon transfer probability is made by fragments with $Z_l \approx 42$. The average probability of observing the muon on the light fragment (calculated with the experimental distribution of the fragments in Z as given in Ref. 7), is 1%, and correspondingly 99% for the heavy fragment.

Since $\tau_0 \approx 0.01 \ll 1$, the model should work well. Nonetheless, to verify the applicability of Demkov's formula, we also calculated with a computer the exact solution of a system of coupled differential equations in the two-level approximation with matrix elements H_{ij} in the field of two Coulomb centers.

An analysis of these results, just as in the physical approximations made in the calculation, allow us to assume that the accuracy of the obtained value is determined by a factor not larger than 2, i.e., the sought average probability for the light fragment lies in the range from 0.5 to 2%.

One important application of our result is the estimate of the muon yield upon conversion of gamma rays of a highly excited fragment on the muon. The theoretical value of the coefficient of internal conversion is larger by one order of magnitude for the light fragment than for the heavy one (see Ref. 8 as well as the more accurate tables of muon conversion coefficients in Ref. 2). Our results therefore alter significantly the estimates obtained in Ref. 2 as well as in Ref. 3 for the muon yield per fission. Our results are important also for the calculation of mesic radiation of a fragment following an "upward throw" of a muon.[9] Comparison of the data of Refs. 2, 3, 8, and 9 with experiment can yield valuable information on nuclei remote from the stability region.

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