Contribution to the theory of nuclear-nuclear interactions at high energies

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(Submitted 12 July 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 5, 314–318 (5 September 1978)

An effective method of calculating a large class of nuclear-nuclear interaction processes, independent of the form of the nuclear density, is proposed within the framework of the theory of multiple collisions.

PACS numbers: 24.50. + g, 24.10. – i

The problem of finding the formal connection between the amplitudes of nuclear-nuclear scattering at high energies and the amplitudes $f(q)$ of NN scattering and the characteristics of the nucleon distributions of the nucleons in the colliding nuclei, within the framework of the theory of multiple scattering, is by itself quite simple. However, when it comes to devising a simple technique for calculating the cross sections of nuclear-nuclear interactions, similar to the one used in the theory of hadron-nuclear interactions, no noticeable progress has been made to date.

The approach we propose consists of establishing an explicit functional dependence of the amplitude of elastic AB scattering (A and B are the colliding nuclei) on the thickness functions $T_A(\mathbf{b})$ and $T_B(\mathbf{b})$ of the nuclei A and B in the optical limit with respect to the atomic numbers of the nuclei A and B and in the approximation in which the nucleon distribution in the nuclei is not correlated. It turns out here that

 0021-3640/78/170288-05$00.60

knowledge of the functional $F\{T_A(\mathbf{b}), T_B(\mathbf{b})\}$ is sufficient for the calculation of other important physical characteristics of the AB interaction. In particular, corrections to the AB-scattering amplitude necessitated by the finite atomic numbers of the nuclei A and B, the corrections necessitated by the correlations in the distributions of the nucleons in the nuclei, the amplitude of the excitation of one of the nuclei or of both simultaneously, the cross section of the quasielastic scattering, as well as many other quantities, are all expressed in terms of the functional derivatives of the functional $F\{T_A, T_B\}$ with respect to the thickness functions T_A and T_B.

We describe briefly the procedure for obtaining an explicit expression for the functional $F\{T_A, T_B\}$.

In the optical limit with respect to the atomic number of the nucleus A, the AB-scattering amplitude is given by

$$\Gamma_{AB}(\mathbf{b}) = \frac{1}{2i\pi p}\int F_{AB}(\mathbf{q})\exp(-i\mathbf{q}\mathbf{b})\,d\mathbf{q} = <[1 - \exp[-\int\Gamma_B(\mathbf{b}-\mathbf{s}, s_B)T_A(\mathbf{s})d\mathbf{d}]> \tag{1}$$

where

$$\Gamma_B(\beta, s_B) = 1 - \prod_{i=1}^{B}[1 - \gamma(\beta - \mathbf{s}_B)], \quad s_B = \{s_{B_i}\},$$

$$\gamma(\mathbf{b}) = \frac{\tilde{\sigma}}{4\pi a_0}\exp-\left(\frac{\mathbf{b}^2}{4a_0}\right), \quad \tilde{\sigma} = \sigma(1 - ia), \quad a = \frac{\mathrm{Re}\, f(0)}{\mathrm{Im}\, f(0)},$$

a_0 is the slope parameter of the elastic NN scattering amplitude. The angle brackets denote averaging over the wave functions of the nucleus B. For example, for an arbitrary function

$$g(s_B): \; < g(s_B) > = \int g(s_B)[\prod_{i=1}^{B} T_B(\mathbf{s}_{B_i})\,d\mathbf{s}_{B_i} / B].$$

Applying to expression (1) the rule for averaging exponentials:

$$<\exp(x)> = \exp\{<x> + \frac{1}{2!}<(x - <x>)^2> + \frac{1}{3!}<(x - <x>)^3> + \dots\}$$

going from the optical limit with respect to the atomic number B, and assuming the NN-scattering radius ($\sqrt{a_0}$) to be small compared with the radii $R_{A,B}$ of the nuclei A and B, we obtain for the phase function $\chi(\mathbf{b})$, accurate to terms of order $a_0/R^2_{A,B}$, the expression:

$$-\chi(\mathbf{b}) = \ln(1 - \Gamma^{AB}_{\mathrm{opt}}(\mathbf{b})) = \frac{2}{\tilde{\sigma}}\int d\mathbf{s}\,\{x[\exp(-y) - 1]$$

$$+ \frac{x^2}{2!}(y + \tfrac{1}{2}\epsilon y^2 + \tfrac{2}{9}\epsilon^2 y^3 + \dots)\exp(-2y) + \frac{1}{3!}x^3(-y + 3y^2 + \tfrac{2}{3}\epsilon y^2 + \dots)$$

$$\times \exp(-3y) + \dots\}, \tag{2}$$

where

$$x = \frac{\bar{\sigma}}{2}\, T_A(\mathbf{b}), \quad y = \frac{\tilde{\sigma}}{2}\, T_B(\mathbf{b} - \mathbf{s}), \quad \epsilon = \frac{\tilde{\sigma}}{16\pi a_0}.$$

The order n of the quantity ϵ in (2) determines the number of closed loops in the diagrams that represent the quantity $\chi(\mathbf{b})$ (see Fig. 1; a—diagram without closed loops $\sim\epsilon^0$, b—diagram with one closed loop $\sim\epsilon^1$). It can be shown that the diagram with closed loops that have rather complicated analytic structure, make in most cases of practical interest a negligible contribution to the measured quantities, compared with the diagrams that contain no closed loops. In this paper we confine ourselves to presenting an expression for the phase function $\chi(\mathbf{b})$ in an approximation wherein only terms of order ϵ^0 and ϵ^1 are taken into account in (2). The result of the summation of the series (2) can be written in the form[3]

$$\chi(\mathbf{b}) = z(e^u - 1) + u(e^z - 1) - uz + 0(\epsilon^2), \tag{3}$$

where

$$u = xe^{-z}, \quad z = ye^{-u}.$$

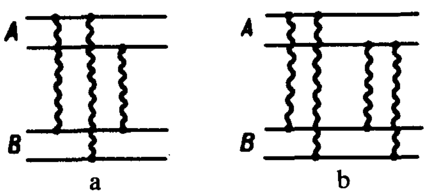

FIG. 1.

Having now expression (3) for the phase function we can, as already mentioned, calculate a number of other quantities. Thus, the corrections of order $1/A$ (or $1/B$) to the amplitude of elastic AB scattering are given by

$$\delta_{A,B}F = \frac{1}{2A(B)} \int T_{A(B)}(\mathbf{s}_1)\, T_{A(B)}(\mathbf{s}_2) \frac{\delta^2 F\{T_A, T_B\}}{\delta T_{A(B)}(\mathbf{s}_1)\, \delta_{A(B)} T(\mathbf{s}_2)}\, d\mathbf{s}_1 d\mathbf{s}. \tag{4}$$

The corrections due to pair correlations of the nucleons in the target nucleus A are

$$\delta F = \frac{1}{2} \int C^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \frac{\delta^2 F\{T_A, T_B\}}{\delta T_A(\mathbf{s}_1)\, \delta T_A(\mathbf{s}_2)}\, d\mathbf{s}_1 d\mathbf{s}_2 dz_1 dz_2, \tag{5}$$

where $C^{(2)}(\mathbf{r}_1,\mathbf{r}_2)$ is the correlation function.

The amplitude for the excitation of nucleus A from the state i into the state f by the nucleus B, i.e., of the reaction $B + A_i \rightarrow B + A_f$, takes in the approximation of one inelastic collision the form

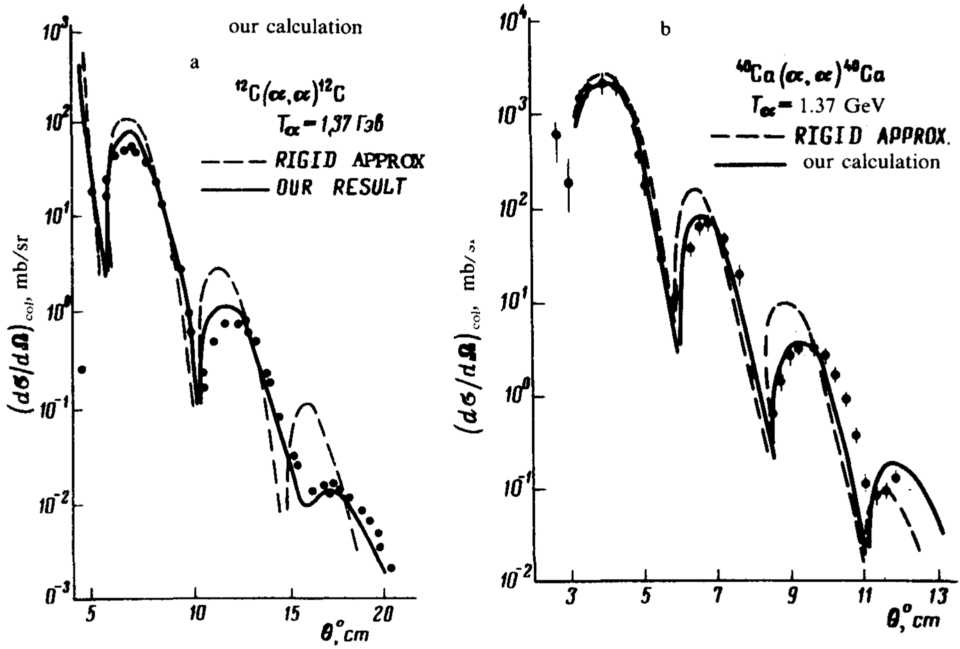

FIG. 2.

$$F_{if} = \int \rho_{if}(\mathbf{r}) \frac{\delta F\{T_A, T_B\}}{\delta T_A(\mathbf{s})} d\mathbf{s}\, dz, \tag{6}$$

where $\rho_{if}(\mathbf{r})$ is the so-called transition density.

The cross section of the quasielastic scattering of the nucleus B by the nucleus A (the nucleus B remains in the ground state, the nucleus A undergoes all kinds of possible excitations, including disintegration), is given by

$$\frac{d\sigma}{d\Omega} = \sum_{k=1}^{A} \frac{1}{k!} \int T_A(\mathbf{s}_1)\ldots T_A(\mathbf{s}_k) \left| \frac{\delta^k F(T_A, T_B)}{\delta T_A(\mathbf{s}_1)\ldots\delta T(\mathbf{s}_k)} \right|^2 d\mathbf{s}_1\ldots d\mathbf{s}_k. \tag{7}$$

Figures 2 and 3 show the results of the cross sections of elastic and inelastic (solid curves) scattering of α particles by ^{12}C and ^{40}Ca nuclei. The wave functions of the interacting nuclei were calculated in the harmonic-oscillator model. Account was taken of effects connected with the motion of the mass center of the nuclei. For comparison, we show the results of calculations by the model of a rigid incident nucleus[2] (dashed curve) and by the optical model of Ref. 3 (dash-dot curve). The experimental data were taken from Ref. 4. A detailed derivation of expression (3) for the phase function, as well as details of the calculations and estimates of the contributions of various correction effects, will be published later.

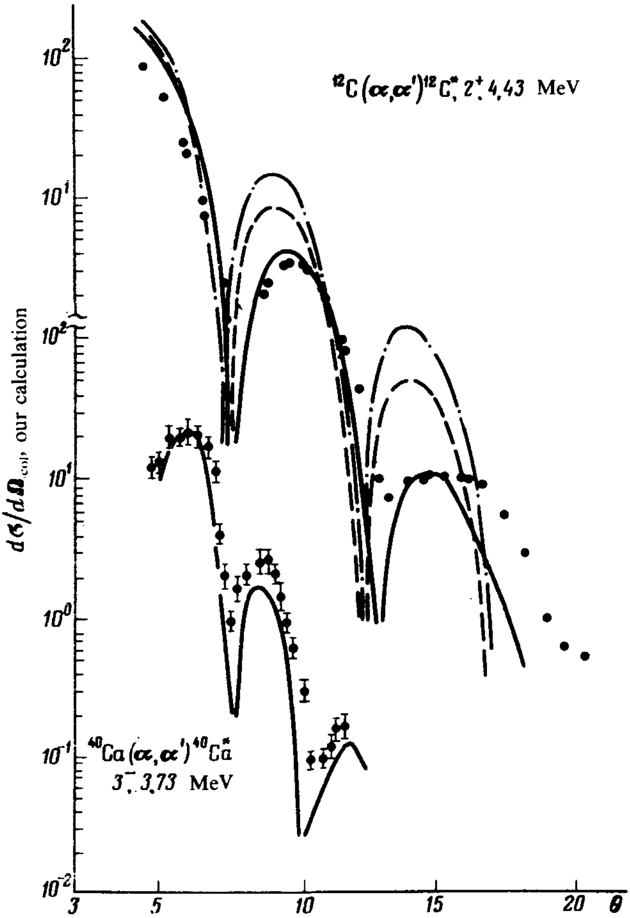

FIG. 3.

The authors thank I.V. Andreev, L.I. Lapidus, and L.N. Strunov for interest in the work and for useful discussions.

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[3)]The integral expression in (3) was first obtained in the form of an expansion in a double series in powers of x and y by Andreev,[1] who used the generating-function method.

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[1]I.V. Andreev and P.N. Lebedev, Physical Institute Preprint No. 92, 1976, Moscow.
[2]G.D. Alkhazov, T. Bauer *et al.*, Nucl. Phys. A **280**, 365 (1977).
[3]W. Czyz and L.C. Maximon, Ann. Phys. **52**, 59 (1969).
[4]A. Chaumeaux, G. Bruge *et al.*, Nucl. Phys. A **267**, 413 (1976).