Mass spectrum and Regge trajectories of light vector mesons in the relativistic quark model

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The mass spectrum and the Regge trajectories of vector mesons are calculated on the basis of the relativistic two-particle quasipotential equation with a barrier potential that increases linearly with distance.

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An analysis by Barbieri *et al.*[1] has shown that the model successfully used to described the charmonium spectrum, based on a Schrödinger equation with a linearly growing potential $V(r) = \sigma r$, is not self-consistent when used for the description of the spectrum of light vector mesons such as ρ, ω, or ϕ. It has turned out that the contribution of the relativistic contribution is comparable with the contribution of the nonrelativistic Hamiltonian chosen to be as the principal one.

In the present paper we use the Kadyshevskiĭ relativistic two-particle quasipotential equation, which takes the following form, in the relativistic configuration representation introduced in[2], for the wave function of the relative motion of two quarks

$$\left[\mathrm{ch}\left(i\lambdabar\frac{\partial}{\partial r}\right)+\frac{i\lambdabar}{r}\,\mathrm{sh}\left(i\lambdabar\frac{\partial}{\partial r}\right)+\frac{\lambdabar^2 l(l+1)}{r^2}\,\exp\left(i\lambdabar\frac{\partial}{\partial r}\right)+\frac{V(r)-W}{2mc^2}\right]\Psi_l(r)=0. \tag{1}$$

Here $\lambdabar = \hbar/mc$, m is the mass of the quark, W is the total energy of the system of two particles, i.e., the meson mass, and the quasipotential $V(r)$ is chosen in the form $V(r) = \sigma r^s$, with $s > 0$. This potential satisfies the condition for the validity of the quasiclassical method developed in[3] for the solution equation. In nonrelativistic quantum mechanics there exists a "modified" quasiclassical method[4] that makes it possible, for potentials of the type $V(r) = \sigma r^s$, to separate the contribution of the centrifugal term.[5] Repeating the analogous transformations for the relativistic case, we arrive at a "modified" quantization condition

$$\int_{r_-}^{r_+} dr\,\ln\left[\frac{W-V(r)}{2mc^2}+\sqrt{\left(\frac{W-V(r)}{2mc^2}\right)^2-1}\right]=\lambdabar\pi\left(n+\frac{l}{2}+\frac{3}{4}\right), \tag{2}$$

that makes it possible to obtain in simple fashion the mass spectrum and the Regge trajectories of relativistic bound systems. For potentials of the type $V(r) = \sigma r^s$ we get from (2)

$$\sqrt{\frac{\pi}{2}}\left(\frac{2mc^2}{\sigma}\right)^{1/s}(\mathrm{sh}\,\chi)^{1/2+1/s}\,\Gamma\left(1+\frac{1}{s}\right)P_{-1/2}^{-1/2-1/s}(\mathrm{ch}\,\chi)=\lambdabar\pi\left(n+\frac{l}{2}+\frac{3}{4}\right), \tag{3}$$

where $\chi = \mathrm{Arch}(W/2mc^2)$ and $P_\nu^\mu(z)$ are Legendre functions.

We consider now the family of $\rho(\omega)$ mesons.[1] it is customarily assumed that the first radial excitation $\rho(0.773)$ is the resonant structure observed in some experiments in the region of 1.25 GeV. The corresponding particle masses obtained for $s = 1$ with the aid of (3) differ insignificantly from the masses obtained in[6] by solving Eq. (1) with $l = 0$ by a numerical method.

Another interesting possibility is therefore the choice, as the first radial excitation of the $\rho(\omega)$ meson the vector meson of mass 1.110 GeV observed at DESY[7] and predicted earlier on the basis of a group-theoretical method of constructing the barrier potentials.[8] The results given by (3) for this case are listed in Table I.

TABLE I. Mass spectrum of vector mesons at the model parameters $\sigma = 0.031$ GeV^2 and $m = 0.18$ GeV.

	$l = 0$		$l = 1$	
n	M^{theor}, GeV	M^{exp}, Gev	M^{theor}, GeV	M^{exp}, GeV $I^G(J^P) = 1^+(1^-)$
0	0.773	$\rho(\omega)$ (0.773)	0,951	δ (9,97)
1	1.110	[7] (1.110)	1,256	not observed
2	1,394	[9] (1,384)	1,525	F_1 (1.54)
3	1.651	ρ'' (1.60) ω'' (1,667)	1.773	x (1,795)
4	1.891	not observed	2,228	u (2,36)
5	2,118	ρ (2,1)	——	——

The parameters of the model are the following: $\sigma = 0.031$ GeV^2 and $m = 0.18$ GeV. The factor $\chi/\sinh\chi$ serves as a measure of the "relativism" of the system, and ranges in the case of the levels listed in the table from 0.7 (for $n = 0.1$) to 0.4[2] (for $n = 4.5$).

An attractive feature of this model with a first radial excitation 1.110 GeV is the proximity of the mass 1.394 (second radial excitation) to the mass of the observed vector resonant structure in the region of 1384 ± 8 MeV.[9] In the usual scheme with $\rho'(1.250)$ this level does not appear, and the level that follows $\rho'(1.250)$ is close to the mass of the $\rho''(1.6)$ meson. In the considered system with $\rho'(1.110)$, the $\rho''(1.6)$ meson enters as the third radial excitation of the ρ meson.

An essential result of the use of the relativistic formalism for the description of light vector mesons is that in the potential $V(r) = \sigma r$ the Regge trajectories are straight

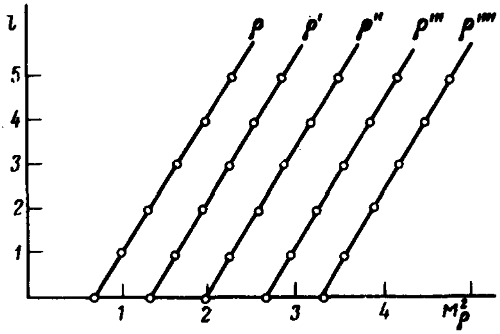

FIG. 1. Regge trajectories of ρ mesons for the potential $V(r) = \sigma r$.

lines, with good accuracy, in the considered mass interval (see Fig. 1). We recall that in nonrelativistic theory straight-line trajectories appear only in an oscillator potential.

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[1] Since the ρ and ω mesons have close masses, we obtain close results for the meson family. They will be presented in a more detailed paper, together with a description of the spectra, the trajectories, and the lepton widths of the Ψ and γ mesons.

[2] This factor is close to unity for mesons with almost nonrelativistic internal motion.

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