# Singularities of magnetic phase transitions in heavy rare-earth metals

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The magnetic phase transitions in the heavy rare-earth metal thullium (Tm) were investigated on the basis of the fluctuation theory of phase transitions. It is shown that $N+1$ ($N=6$) stationary states are produced in thullium when it goes from the paramagnetic region into the disordered state, and only in the lowest order state does the crystal have a nonzero spontaneous magnetic moment. The phase transition to the ground state of the system turns out to be of second order, and all others are fluctuation phase transitions of first order.

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Experimental investigations of magnetic phase transitions in heavy rare-earth metals[1,2] have shown that in holmium and erbium there exist, in addition to ordinary planar spiral (*NS*), longitudinal sinusoidal wave (*c*-sin), and conical spiral states, also a tilted spiral (*TS*) state. It follows from the dependence of the specific heat on the temperature that this corresponds to the appearance of an additional peak. Phase transitions in rare-earth metals and their compounds were investigated in detail in Refs. 3 and 4, where good agreement with experiment was obtained.

It was also established by experiment[2] that the magnetic structure of thullium in the lowest ordered state is a ferrimagnetic antiphase domain with strong anisotropy along a preferred axis of the crystal (Fig. 1).

In this communication we consider the magnetic phase transitions in thullium. The magnetic structure of thullium[1)] (space group $P6_3/_{mmc} - D^4_{6h}$) can be represented as a superposition of $N+1$ longitudinal sinusoidal waves with periodicity along the preferred $z$ axis of the crystal (hexagonal axis). It can be set in correspondence with $N+1$ spin-density vectors

$$\mathbf{S}_0,$$

$$\mathbf{S}_1 = \mathbf{s}_1^+ + i\mathbf{s}_1^-, \quad \mathbf{S}_2 = \mathbf{s}_2^+ + i\mathbf{s}_2^-, \ldots, \quad \mathbf{S}_N = \mathbf{s}_N^+ + i\mathbf{s}_N^-, \tag{1}$$

where $\mathbf{S}_0$ is the ferromagnetism vector, since the system has a nonzero magnetic moment in the lowest ordered state.

Having a set of vectors $\mathbf{S}_0$ and $\mathbf{S}_i$ we can determine the Landau free energy in the paramagnetic region. For the case of a real crystal with large anisotropy we obtain

  

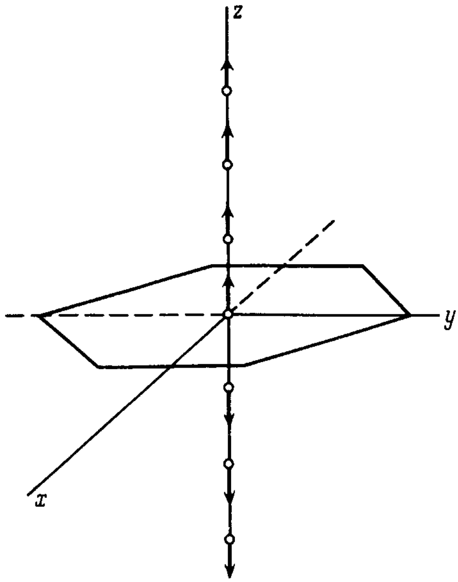


FIG. 1. Magnetic structure of thullium in the ground state.

$$F = \frac{1}{2} r_0 s_0^2 + \frac{1}{2} \sum_{i=1}^{N} r_i (s_i^{+2} + s_i^{-2}) + \frac{1}{8} \Gamma_0 s_0^4 + \frac{1}{8} \sum_{i=1}^{N} \Gamma_i (s_i^{+2} + s_i^{-2})^2$$

$$+ \frac{1}{4} \sum_{i=1}^{N} \Gamma_{0i} s_0^2 (s_i^{+2} + s_i^{-2}) + \frac{1}{8} \sum_{\substack{i=1,\, j=1 \\ i \neq j}}^{N} \Gamma_{ij} (s_i^{+2} + s_i^{-2})(s_j^{+2} + s_j^{-2}), \tag{2}$$

and the bare values of the temperatures and the amplitudes satisfy the following relations:

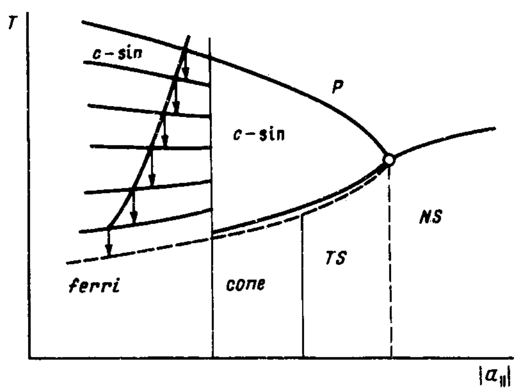


FIG. 2. Phase diagram of rare-earth metals and their alloys.

$$\tau_0^\circ < \tau_1^\circ < \ldots < \tau_N^\circ, \quad \tau_0^\circ \lesssim \tau - |a_{\parallel 0}|, \quad \tau_i^\circ = \tau - |a_{\parallel i}|,$$

$$\Gamma_1^\circ < \Gamma_2^\circ < \ldots < \Gamma_N^\circ, \quad \Gamma_{01}^\circ = \Gamma_{02}^\circ = \ldots = \Gamma_{0N}^\circ, \quad \Gamma_{ij}^\circ = \Gamma_{i'j'}^\circ, \tag{3}$$

$|a_{\parallel i}|$ is the modulus of the axial anisotropy of the $i$-th harmonic.[2)]

On moving from the paramagnetic region, the most strongly fluctuating is the field of the ferromagnetic vector $\mathbf{S}_0$, and the natural fluctuations of the first-harmonic vector $\mathbf{S}_1$ are weaker. Using the renormalization-group ($RG$) formalism, we can obtain equations for the amplitudes and temperatures of the zeroth and first harmonics from Ref. 4. It follows from the solution of these equations that the first harmonic becomes unstable as the result of its receiving the energy of the ferromagnetic-vector field fluctuation, and the system undergoes a first-order phase transition into a state with a longitudinal sinusoidal wave. Following the transition into a ($c$-sin) state, the anisotropy of the system decreases for example, the anisotropy of the "ferromagnetic component" decreases by an amount $\frac{1}{4}\Gamma_{01}(x_{0s})s_{10}^2$, where $s_{0s}$ is the value of the $RG$ method at the point of the transition into the ($c$-sin) state, while $s_{10}$ is the jump of the order parameter of the first harmonic. The change of the anisotropy of the remaining harmonics preserves the inequalities between the temperatures $\tau_0^0 < \tau_2^0 < \ldots < \tau_N^0$ [see (2)].

By analogous reasoning it can be shown that the system will undergo first-order transitions from one stationary state to another stationary state with a modulated moment along the $z$ axis while the transition to the main ordered state will be of second order. Consequently, $N+1$ stationary states appear on the $(T, |a_\parallel|)$ diagram (the number of the states for thullium is 7, since the period of the magnetic structure is equal to $7a$, where $a$ is the lattice constant in the direction of the $z$ axis) (see Fig. 2). We can thus expect a number of additional peaks to appear on the plot of the specific heat against the temperature in the interval between the ($c$-sin) and (ferri) states.

1)Similar investigations can be carried out also for other substances with similar magnetic structures.
2)The remaining symbols are the same as in Ref. 4.


1K.P. Belov, M.A. Belyanchikova, R.Z. Levitin, and S.A. Nikitin, Redkozemel'nye ferro-*i* antiferromagnetiki (Rare-Earth Ferro- and Antiferromagnets), Nauka, 1965.
2A.H. Milhouse and W.C. Koehler, Int. J. Magnetism **2**, 389 (1971); Magnetic Properties of Rare Earth, ed. by R.J. Eliott, New York, Plenum, 1972.
3I.E. Dzyaloshinskiĭ, Zh. Eksp. Teor. Fiz. **72**, 1930 (1977) [Sov. Phys. JETP **45**, 1014 (1977)].
4M.A. Savchenko and A.V. Stefanovich, Zh. Eksp. Teor. Fiz. **74**, 2300 (1978) [Sov. Phys. JETP **47**, 1195 (1978).