# Destabilizing effect of shear and maximum pressure of plasma in a tokamak

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An analytic criterion is obtained for the stability of a plasma to ballooning modes. It takes into account two new physical effects, namely destabilization due to crossing of the flute-oscillation branches, and the balloon effect due to the shear.

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In the present paper we investigate, with the aid of Taylor's method[1] the small-oscillation equation that describes flute instability and ballooning effects. A new analytic stability criterion is obtained for realistic tokamak geometry. This criterion shows that with increasing shear the stability first deteriorates and then improves again. The limiting values of $\beta_J$ (above which instability sets in), which follow from this criterion, can be less than unity.

Small-scale flute oscillations were investigated theoretically in many papers. In Refs. 2 and 3 they obtained an expression for the stability criterion of flute oscillations in a tokamak with circular magnetic surfaces. The necessary stability criterion of Ref. 3 is of the form

$$\frac{1}{4}s^2 + \frac{2p'r}{B_s^2}(1-q^2) > 0. \qquad (1)$$

It was also shown in Ref. 3 that expression (1) follows from the general geometric criterion of Mercier.[4]

It will be shown below that the plasma can be unstable even if criterion (1) is satisfied.

To describe flute instability at large $n$ ($n$ is the azimuthal number along the major azimuth of the torus) we use a simplified variational principle that is valid in the case of a strong longitudinal magnetic field.[5] Choosing in accordance with Ref. 1 the electrostatic potential in the form $\phi(\rho,\theta)=F(\rho,\theta)e^{inq\theta}$, and changing the integration variables $\int_0^{2\pi}d\theta\ldots\rightarrow\int_{-\infty}^{+\infty}dy\ldots$, we obtain the following expression for the potential energy in the limit $(nq)\gg 1$:

$$W_0 = \frac{1}{2}\int_0^a d\rho (nq)^2 \int_{-\infty}^{\infty} dy \left\{ \frac{1}{4\pi\sqrt{g}}\left(\frac{B^\omega}{B^s}\right)^2 [g_{11} - 2g_{12}(sy) + g_{22}(sy)^2]\left(\frac{\partial F}{\partial y}\right)^2 \right.$$

$$\left. + \frac{1}{\sqrt{g}B^s}\frac{dp}{d\rho}\left(\frac{\partial}{\partial\rho}\frac{1}{B^s} - (sy)\frac{\partial}{\partial y}\frac{1}{B^s}\right)(F)^2\right\}, \quad s = rq'/q, \qquad (2)$$

  

where the expressions for $g_{ik}, g^{1/2}$ and $B^s, B^w$ are given in Ref. 3.

By varying (2) with respect to $F$ we easily obtain a small-oscillation equation that describes the flute instability with ballooning effects taken into account. This equation differs from the corresponding equation of Ref. 2 in that it is written in a coordinate system with straight force lines ($q$ is independent of $y$).

To obtain an analytic criterion, we use a method that generalizes the averaging method in the theory of nonlinear oscillations. For small values of $s$, the solution of the equation for $F$ can be represented in the form $F = \langle F \rangle + \tilde{F}$, where $\langle F \rangle$ is the slowly varying part (the characteristic variation interval is $y \sim 1/s$) and $\tilde{F}$ is the rapidly varying part of the solution (interval $y \sim 1$). We can obtain for $\tilde{F}$ an expression of the type

$$\tilde{F} = \frac{A \cos y + B sy \sin y}{1 + (sy)^2} \langle F \rangle , \qquad (3)$$

where $A$ and $B$ are functions of $g_{11}$ and $g_{12}$, etc. After substituting (3) in the equation obtained for $F$ by varying (2), and averaging the latter over the fast oscillations, we can obtain an equation for $\langle F \rangle$. This approximation corresponds to the averaging method, but we shall proceed in a different manner, namely, we substitute a solution of the type $F = \langle F \rangle + \tilde{F}$ directly in the expression for the potential energy (2). We can assume here that $\langle F \rangle$ is approximately constant with respect to $y$ (Ref. 6). From the condition $W_0 > 0$ we immediately obtain the *necessary* local stability criterion. It can be shown that this method is much more accurate than the direct solution of the equation for $\langle F \rangle$ [(the latter leads after a number of simplifications to the criterion (1)].

This method is more accurate, first because the trial function we substitute differs little from the exact function $F = \langle F \rangle + \tilde{F}$, where $\tilde{F}$ is given by expression (3), obtained by asymptotic expansion in powers of $s$), and it is well known that in this case the accuracy of the answer (increases rapidly on account of the variational principle; second, we obtain in this manner (as will be shown below) exponential non-analytic terms (of the type $e^{-1/s}$), which are in principle not taken into account in the asymptotic method of averaging.

The criterion obtained for a realistic geometry[3] takes the following form

$$\frac{1}{2} s^2 + \frac{2p' r}{B_s^2} \left\{ 1 - q^2 \left[ 1 - \frac{7}{4\epsilon} \left( 1 - \frac{5}{7} s^2 \right) e^{-1/|s|} \right] \right\} - 3 \frac{s q^4}{\epsilon^2} \left( \frac{p' r}{B_s^2} \right)^2 > 0, \qquad (4)$$

where $\epsilon = r/R$.

This criterion is shown in the Fig. 1 for different values of $\epsilon$. Figure 1 shows the curves that follow from Ref. 1. The region to the right of the curves is unstable. We emphasize once more that in this case the criterion (1) is satisfied, i.e., the plasma is stable in the sense of Mercier.

The tokamak geometry used by us is similar in the limit as $\epsilon \to 0$ to the Taylor metric. It is therefore not surprising that it agrees well with the numerical results.

It is clearly seen from our curves that as $s \to 0$ the plasma is more stable—this is a manifestation of the stabilizing influence of the magnetic well. With increasing shear, the stability becomes worse (!). The destabilization is due to the new terms $e^{-1/|s|}$ and $s(p'r/B_s^2)^2$. The effect described by the exponential term $-e^{-1/|s|}$ was not taken into account before. It is analogous to the effect of level splitting of atoms as they approach a molecule. The destabilizing term $s(p'r/B_s^2)^2$ characterizes the balloon effect with decreasing current. With further increase of the shear, stability is restored.

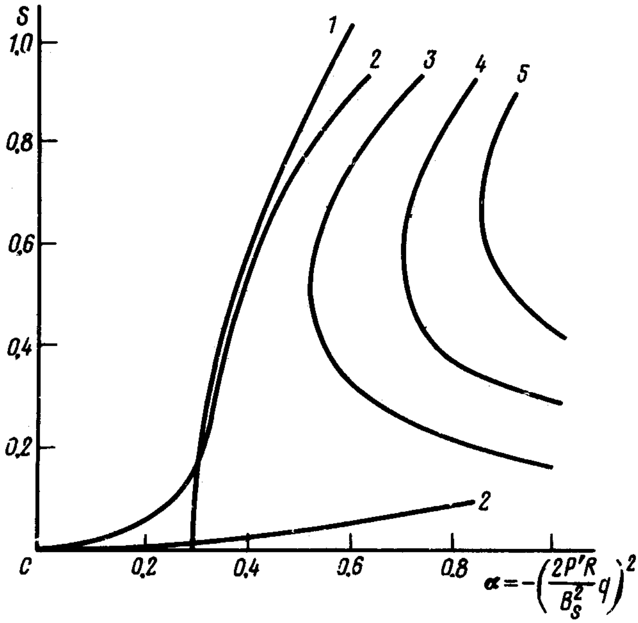


FIG. 1. 1—From Ref. 1; 2 — $\epsilon = 0$; 3 — $\epsilon = 1/10$; 4 — $\epsilon = 1/5$; 5 — $\epsilon = 1/3$.

At $\epsilon = 0$ the stabilization by the magnetic well vanishes (curve 2) and instability is possible starting with $\alpha = 0$. It follows from these curves that the limiting plasma pressure can be quite small, $\beta_J \lesssim 1$.

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