New approach to the hydrodynamic theory of multiple production of hadrons

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An approach that makes it possible to find the dynamic connection between the characteristics of the leading particles and the behavior of the spectra of secondary pions is developed on the basis of scaling solutions of the hydrodynamic theory of multiple production. Explanations are offered for the experiments on long-radius correlation forces, and a correspondence is noted between the considered space-time picture of multiple processes and the quark-parton concepts.

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In our preceding papers[1,2] we presented a correct mathematical formulation of scale-invariant solutions in the hydrodynamic theory of multiple processes. It was shown that the scaling requirement necessitates the introduction of particle-light objects on the boundaries between the hadron liquid and the vacuum. The complete energy-momentum tensor $\Gamma^{\mu\nu}$ of the hadron system (integration over its cross section) is then represented in the form

$$\Gamma^{\mu\nu} = \frac{\pi}{\mu^2}[(\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}]\theta(x - x_1(t))\theta(x_2(t) - x) + \sum_{k=1,2} \frac{m_k(t)\,\dot{x}_k^\mu \dot{x}_k^\nu}{(1 - \dot{x}_k^2)^{1/2}}\,\delta(x - x_k(t)), \qquad (1)$$

where μ is the pion mass, m_k are the masses of the boundary particles, ϵ is the energy density of the liquid, $p = c_0^2\epsilon (0 \leqslant c_0^2 < 1)$ is the pressure, u^ν is the 4-velocity, and x_1 and x_2 are the coordinates of the boundary.

The equations of motion of the hadron system take the form of conservation laws

$$\partial \Gamma^{\mu\nu}/\partial x^\nu = 0\,. \qquad (2)$$

They describe the evolution of both the liquid and of the boundary particles. By way of a hydrodynamic solution inside the liquid we use the scaling solution[1-3]

$$u^\mu = \frac{x^\mu}{\tau}\left(v = \frac{x}{t}\right) \qquad \epsilon = \epsilon^*\left(\frac{\tau^*}{\tau}\right)^{1+c_0^2}, \qquad (3)$$

where $\tau = (t^2 - x^2)^{1/2}$ is the proper time of the element. The quantities marked by asterisks correspond to the instant of the decay of the liquid into secondary hadrons ($\epsilon^* \approx \mu^4$, and τ^* is a parameter of the model).

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When (3) is taken into account, the equations of the dynamics of the boundary have exact analytic solutions[1,2]

$$(-1)^k[E_k x_k(t) - P_k t] + \frac{\pi}{\mu^2}\,\frac{\epsilon^* \tau^{*\,1+c_0^2}}{1-c_0^2}\,\tau_k^{1-c_0^2} = Q_k\,, \tag{4}$$

$$m_k^2(t) = E_k^2 - P_k^2 + \frac{\pi^2}{\mu^4}\,\epsilon^{*2}\tau^{*\,2(1+c_0^2)}\,\frac{1+c_0^2}{1-c_0^2}\,\tau_k^{-2c_0^2} - 2\frac{\pi}{\mu^2}\,\epsilon^*\left(\frac{\tau^*}{\tau_k}\right)^{1+c_0^2} Q_k\,, \tag{5}$$

and in the asymptotic limit as $t \to \infty$ we have

$$m_k = \sqrt{E_k^2 - P_k^2}\,, \tag{6}$$

where E_k, P_k, and Q_k are the integration constants (in the c.m.s.)

$$E_1 + E_2 = \sqrt{s}\,,\ \ P_1 + P_2 = 0);\qquad \tau_k \equiv (t^2 - x_k^2(t))^{1/2}.$$

Equation (2) on the boundaries describes the energy and momentum exchange between the liquid and the particle-like objects. On the surface $(t^2 - x^2)^{1/2} = \tau^*$ where the secondary hadrons are produced we identify these objects with the leading particles.

It follows from (4) that the stage of hydrodynamic expansion begins with a certain nonzero instant of time. It is natural to assume that prior to that instant the quantum effects play a substantial role.[4] The initial conditions of the hydrodynamic expansion are formed during the quantum stage and have a leeway that is reflected in our case in the presence of the integration constants E_k, P_k, and Q_k and the parameter τ^*. To calculate the average characteristics of the secondary particles it is necessary thus to average over the possible initial conditions. For this purpose it is convenient to connect the quantities E_k, P_k, Q_k and τ^* with the measured characteristics of the process. The structure of an individual event in rapidity space is similar to that shown in Fig. 1. Using (3)–(5) we have

$$E_k^M = \frac{\pi}{\mu^2}\,\epsilon^*|x_k(\tau^*)| = \frac{\pi}{\mu^2}\,\epsilon^*\tau^*\,\mathrm{sh}\,|y_k^M| \equiv h\mu_\perp \mathrm{sh}\,|y_k^M|\,, \tag{7}$$

where E_k^M is the energy of the mesons from the decay of the liquid, $\mu_\perp$ is the average "transverse mass" of the secondary pions ($\mu_\perp \approx 0.4$ GeV). We assume formula (6) for the mass of the leading particles and replace m_k in the calculations by the effective value $\bar{m}_k \equiv m = (1-2)m_p$ (m_p is the proton mass). At $m \leqslant$ const and $s^{1/2} \to \infty$ we have $E_1 \approx E_2 \approx s^{1/2}/2$. Relations (3)–(5) and (7) make it possible, by eliminating E_k, P_k, Q_k,

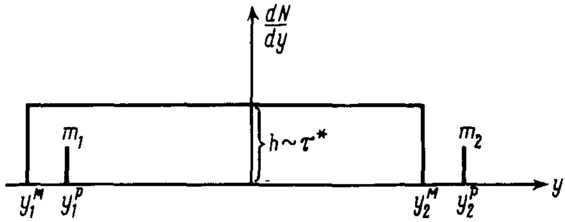

FIG. 1. Rapidity distribution of secondary hadrons in the c.m.s.: $y_{1,2}^P$—rapidities of the leading particles, $y_{1,2}^M$—minimal (maximal) rapidity of pion spectrum from the decay of the liquid, h—height of plateau.

and τ^*, to express the characteristics $y_{1,2}^M$ and h of the meson spectrum directly in terms of the masses and rapidities of the leading particles

$$h(x_0) = \frac{m}{\mu_\perp} \frac{1 - \frac{m}{m_p} x_0}{\sqrt{\frac{m}{m_p} x_0}}, \quad x_0 = \frac{2m_p}{\sqrt{s}} \operatorname{ch} y^P, \tag{8}$$

$$y^M(x_0) = \tfrac{1}{2} \ln \frac{s\, x_0}{m m_p} . \tag{9}$$

According to (8) and (9), averaging over the initial conditions can be replaced by averaging over the rapidities (or x_0) of the leading particles. We ultimately arrive at the conclusion that the process of multiple production in hadron collisions is described by an incoherent sum of contributions with secondary pion spectra that are flat in rapidity, such that the height and width of the plateau changes from collision to collision. We note first that the result of model (8) accounts well for the experimental long-radius correlations between the height of the plateau at the center and the proton momentum.[5] Relation (8) reflects also the connection between the average height $\bar{h}$ of the plateau, and the mass m of the leading particle. The increase of the height of the plateau with increasing energy, observed at contemporary energies, is attributed by us by the same token to the growth of the average mass of the leading particles (clusters) that are produced in the collisions.

We consider now the multiplicity distribution for one of the halves of the system. At a fixed x_0 we have a pion spectrum that is flat in the rapidity, with a height $h(x_0)$ (8) and a length $y^M(x_0)$ (9). As usual in the hydrodynamic approach, we assume a Poisson multiplicity distribution for these events. The resultant distribution is then obtained in the form

$$\mathcal{P}_N = \int_{2m_p/\sqrt{s}}^{x_{0max}} \frac{[y^M(x_0)h(x_0)]^N}{N!} e^{-y^M(x_0)h(x_0)} \frac{1}{\sigma} \frac{d\sigma}{dx_0} dx_0 . \tag{10}$$

The spectrum of the leading proton is chosen in the simplest form $(1/\sigma)(d\sigma/dx_0) \pm 1$, which does not contradict the experimental data and excludes from our consideration the inessential region $x_0 \approx 1$. At $N > \ln(s^{1/2}/m)$ the integral (10) can be calculated by the saddle-point method (the saddle point moves towards lower limits with increasing N); this yields

$$\mathcal{S}_N \approx \frac{2mm_P}{\mu_\perp^2 \bar{h}^2} \frac{1}{N}\left(\frac{\bar{N}}{N}\right)^3 , \tag{11}$$

where $\bar{N} \sim \ln(s^{1/2}/m)$ is the average number of particles for half the system. We call attention to the fact that the distribution (11) satisfies the *KNO* scaling,[6] and the scaling function $\Psi(Z) \propto Z^{-3}$ itself agrees well with the experimental data at $Z > 1$ (i.e., $N > \bar{N}$).

The model predicts also the existence of positive correlations between the numbers of the particles emitted in the front and rear hemispheres. These predictions reflect in integral fashion that the model incorporates long-range correlations that result from the summation of contributions with different heights of the plateaus. If the number of particles n_F with $y > 0$ is fixed, then the effective contributions, out of the entire sum, are those with plateau heights that give an average particle number close to n_F. According to (8) and (9), the height of the plateau increases with increasing n_F in these events, and the number of particles n_B with $y < 0$ increases. This result agrees with the experimental data.[7]

The space–time pattern of the hadron–hadron interaction developed in the model has a natural quark–parton interpretation. When the high-energy hadrons collide, the valent quarks of the colliding particles pass through each other, leaving behind them a loop of gluons with a flat rapidity spectrum.[8] These valent quarks correspond in our model to particle-like objects on the boundary of the gluon liquid. We establish that the stage of hydrodynamic expansion of such a system sets in at a certain nonzero instant of time after the collision. The parameter τ^* of the model corresponds to the proper lifetime of the partons in the individual collision act. The time of appearance of the secondary pion turns out to be proportional to its energy.

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