Behavior of transport coefficients at relativistic temperatures

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It is shown qualitatively that allowance for the dissipative effects can turn out to be substantial when it comes to describing earlier stages of the evolution of the universe and in the hydrodynamic theory of multiple production of particles.

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At high density and high temperature of matter, dissipative effects can turn out to be substantial in many cases.[1-3] In a state of weak nonequilibrium, it is possible to describe them in terms of transport coefficients (*TC*), the calculation of which in the general case is a complicated problem.

The situation simplifies considerably in the ultrarelativistic limit.[1,4] To this case, a connection was established in Refs. 4 between the temperature behavior of the *TC* and the form of the interparticle interaction, under the assumption that the latter is weak and is characterized by a single coupling constant. It was shown that the temperature dependence of the *TC* is monotonic and is determined by the dimension of the coupling constant: the *TC* increase like T^3 for a theory with a dimensionless coupling constant and decrease with temperature in power-law fashion for nonrenormalizable theories.

We examine below the changes that take place in the temperature dependence of the *TC* when account is taken of other dimensional parameters (mass, second coupling constant). This makes it possible, in particular, to establish the character of the behavior of the *TC* of the neutrino gas during the earlier stages of the evolution of the universe. It turns out also that in some cases the *TC* exhibit singularities typical of systems in which phase transitions are possible.

We consider an ultrarelativistic system of elementary particles, the interaction in which is polynomial and is characterized by two coupling constants $g_1 = g_2$. In the system of units $c = \hbar = k_B = 1$, we introduce the dimensionless constants $g_i^T = g_i T^{n_i} (i = 1,2)$, where the integers n_i are determined by the degree of nonlinearity of the theory and by the statistics of tthe fields, while T is the temperature of the system. Assume that $g_i^T \ll 1$. Confining ourselves to second-order perturbation theory for a quasi-homogeneous state of the system, we obtain in the relaxation-time approximation, in analogy with Ref. 4 for example, an expression for the viscosity coefficient

$$\eta = a T^3 [b_1 g_1^2 T^{2n_1} + b_2 g_2^2 T^{2n_2} + 2 g_1 g_2 T^{n_1 + n_2}]^{-1}. \qquad (1)$$

Here a and b_i are positive numbers, as can be easily verified by turning off one of the interactions [formula (1) yields then the results of Ref. 4]. Let $n_1 \neq n_2$. If g_1 and g_2 are of the same sign, the temperature dependence of the viscosity is monotonic. If, however, one of the interactions is attracting and the other repelling, then the viscosity coefficient $\eta(T)$ has a maximum at a certain point T_c, in the vicinity of which the fluctuations in the system increase. This situation is typical of second-order phase transitions and is fully analogous to the nonrelativistic case (Ref. 5).[1)]

A possible illustration is the simplest model of a complex scalar field with interaction and with a negative square of the mass $(0<\lambda\ll 1)$

$$L = \partial_\mu \phi^* \partial^\mu \phi + \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 . \quad (2)$$

In our notation $g_1 = \mu^2$, $g_2 = -\lambda$, $n_1 = -2$, and $n_2 = 0$. It follows then from (1) that $T_c = \mu/\lambda^{1/2}$, which agrees in order of magnitude with the critical temperature of the relativistic phase transition in this model.[6]

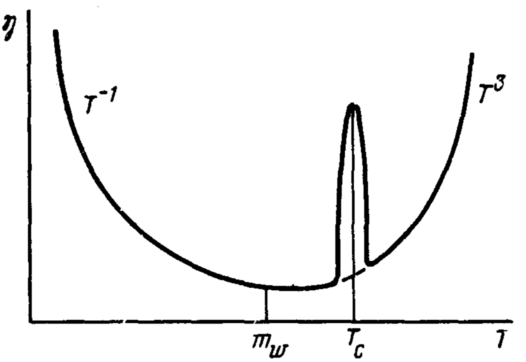

FIG. 1.

We consider now the temperature dependence of the viscosity of a neutrino gas. According to Refs. 4 and 7, the four-fermion model of weak interaction leads to the relation $\eta \propto T^{-1}$. On the other hand, in the Weinberg model, in the ultrarelativistic limit, we have as a result the dimensionless character of the coupling constant $\eta \propto T^3$.[4] There is no contradiction between these results, since they pertain to different temperature regions. If we start from the region $T > m_w$, then the role of the mass of the intermediate boson m_w will increase with decreasing temperature, and this will lead to a derivation from the $\eta \propto T^3$ law. With further decrease of temperature, an instant sets in when the intermediate boson turns out to be "frozen," and the interaction becomes of Fermi contact type with a new coupling constant $G = (2g^2)^{1/2}/m_w$ and a characteristic dependence of the cross section on the energy, which leads to the relation $\eta \propto T^{-1}$ (see Fig. 1). The picture becomes more complicated if account is taken of the relativistic phase transition at the temperature T_c above which the vector-boson mass vanishes.[6] The qualitative behavior of the viscosity of the neutrino gas therefore takes the form shown in Fig. 1.

It was proved in Refs. 3 and 8 that in certain cosmological models allowance for the dissipative properties of the medium can lead to elimination of the singularity.[9]

The necessary condition is a sufficiently rapid growth of the viscosity with temperature, $\eta \propto T^{\alpha}$, where $\alpha > 2$ (Ref. 3). As follows from the foregoing, the viscosity $\eta \propto T^3$ satisfying this condition can take place during the hadron state of the evolution of the universe, either on account of the neutrino component of the medium, or as a consequence of the renormalizability of the quark-gluon interaction in the hadron plasma.[10] We note that the large viscosity of the medium during the earlier stage of the evolution of the universe points also to a fundamental possibility of explaining the anomalously high entropy during the late stages of the expansion.[11]

We note also that the viscosity is large under whose conditions when the particle-production processes become substantial. On this basis we can describe phenomenologically particle production with the aid of viscosity.[3,12] In particular, this point of view may prove useful in the hydrodynamic theory of multiple production of particles, a production that is possibly determined by a phase transition from the quark–gluon state of matter into the hadron state.[10] In the vicinity of the phase transition, the viscosity becomes anomalously large and allowance for it may turn out to be decisive.

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[1)]We point out that this result is only qualitative. A more rigorous analysis of the *TC* in the vicinity of T_c would take us outside the scope of perturbation theory.

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