Does the triton contain a deuteron?

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The peripheral part of the wave function of the triton, which describes its virtual dissociation into a deuteron and a neutron, is obtained. The mean squared radius of the triton calculated with its aid is close to the experimental value.

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The states of the triton t and of the system "deuteron d + neutron n" are separated by a relatively narrow (in the scale of nuclear forces) energy gap $\Delta = E_d - E_t \cong 6$ MeV. Accordingly, the matrix elements of the nd scattering, of the $t \to n + d$ disintegration, etc., have close poles at a momentum transfer $k = iX$, where $X^2 = 4m\Delta'/3$ and m is the mass of the nucleon. Assuming the contribution of such a pole to predominate, Takibaev and the author obtained recently a reasonable value of triton coupling.[1]

From the point of view of the internal structure of the triton, this pole approximation means that the triton stays for an appreciable fraction of the time in the dissociated $d + n$ state. This picture, however, contradicts at first glance the smallness of the triton radius $R_t \cong 1.7$ F compared with the radius of the deuteron itself $R_d = (2^{1/2}\kappa)^{-1} \cong 3$ F where $\kappa^2 = -mE_d$.

The purpose of the present article is to show that this contradiction is in fact illusory. The presence of a third particle deforms (compresses) the deuteron inside the triton to such a degree that the contribution of the deuteron to R_t not only fails to predominate but, on the contrary, is a relatively small fraction. Accordingly, the calculation of R_t with the aid of a wave function taken in the pole approximation leads to a reasonable agreement with experiment.

1. To determine the triton wave function Ψ_t we use a method consisting of describing the evolution of the system with changing coupling constant g (the pair-interaction potential gV) and already used to calculate the binding energy of the triton and the nd-scattering phase shifts.[1,2] The corresponding equation for the wave function is[2]

$$\frac{d\Psi_t}{dg} = \sum_i V_{i,t}\,\Psi_i\,/(E_t - E_i + i\delta)\,, \tag{1}$$

where $V_{i,t}$ is the matrix element of V, i is the index of the intermediate state, which is chosen in the pole approximation to be the $d + n$ state. Introducing the Jacobi coordinates $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\vec{\rho} = \mathbf{r}_3 - (\mathbf{r}_1 + \mathbf{r}_2)/2$ and the wave function of the free deuteron

$$\Psi_d(r) \sim \exp(-\kappa r)/r\,, \tag{2}$$

we have[1)]

$$\Psi_i = \Psi_d(r)\exp(ik\vec{\rho})\,,$$

On the other hand, according to Ref. 1,

$$V_{i,t} \sim \frac{dX}{dg} / (k - iX).$$

Substituting these expressions in (1) and assuming, with logarithmic accuracy, that the ratio κ/X does not depend on g (see Ref. 1), we arrive at the following expression for the wave function of the triton, which is valid on its periphery,

$$\Psi_t \sim \exp(-\kappa r - X\rho) / [r(\kappa r + X\rho)]. \qquad (3)$$

In this region it satisfies the free Schrödinger equation

$$(\Delta_r + \frac{3}{4}\Delta_\rho + mE_t)\Psi_t = 0.$$

2. Expression (3) shows that the deuteron retains its individuality inside the triton [the factor $\exp(-\kappa r)$]. The wave function (3), however, decreases with increasing r at a noticeably faster rate than the wave function of the free deuteron (2). This leads to a considerable suppression of the contribution of the deuteron to the triton radius R_t.

Symmetrization of expression (3) with respect to the coordinates of the particles and calculation of the mean value of $r^2/2 + 2\rho^2/3$ yields

$$R_t^2 = \frac{3}{5(1+b)}[(1+a)/X^2 + 1/8\kappa^2], \qquad (4)$$

where the positive quantities a and b, which are finite at $\kappa = 0$, correspond to exchange (symmetry) effects. Even this shows that the deuteron contribution [the last term of (4)] to R_t is small.

A straight forward but cumbersome calculation, at the real ratio $\kappa/X \cong 1/2$, yields

$$a = 50\ln 3 - \frac{145}{4}\ln 2 - \frac{1865}{72} - \frac{10}{3}I \approx 1.65,$$

$$I = \int_0^{1/3}\frac{dt}{t}\ln\left(\frac{1+t}{1-t}\right), \quad b = 9\left(\ln\frac{3}{2} - \frac{1}{4}\right) = 1.40.$$

Accordingly,

$$R_t \cong 1.9 \text{ F}, \qquad (5)$$

which is quite close to the experimental value given above.

3. The results confirm the concept, arrived at from energy considerations, that the triton is a system consisting of a deuteron and a neutron. This makes it possible, in particular, to use the wave function (3) in the description of the peripheral part of the triton. Of course, the question of the quantitative limits of applicability of the statements made above is still to be investigated.

[1)]The term $-(2\pi/X)^{1/2}\Psi_t/(k - iX)$, which has been omitted here and corresponds to nd interactions, does not influence the asymptotic expression (3).

[1]D.A. Kirzhnits and N. Zh. Takibaev, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 73 (1978) [JETP Lett. **27**, 68 (1978)].
[2]D.A. Kirzhnits and N. Zh. Takibaev, Yad. Fiz. **25**, 700 (1977) [Sov. J. Nucl. Phys. **25**, 370 (1977)].