Photon momentum in a medium with negative group velocity

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The applicability of the expression $\mathbf{p} = n\hbar\omega/c$ for the photon momentum in a refracting medium is discussed in cases of Cerenkov radiation and light refraction by the surface of a medium with negative group velocity (the photon in the medium is regarded as a quasiparticle). The photon momentum and the velocity of its motion are oppositely directed, corresponding to a negative mass of the quasiphoton. The pressure of the light in the medium with negative group velocity should be negative.

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The momentum that must be ascribed to a photon in an ordinary refracting medium is discussed in Ref. 1. Ginzburg,[2] in an analysis of the quantum theory of Cerenkov radiation, was the first to assume its value to be

$$\mathrm{p} = \hbar \mathrm{k} = \frac{n}{c}\hbar\omega , \tag{1}$$

where n is the refractive index of the medium.

Reference 1 considers the classically calculated, on the basis of Refs. 3–5, electromagnetic-field force acting on the translational motion of the particle. The force is such that if the energy and momentum conservation laws are used in the quantum treatment, then in fact we must assume (1). This was done, in particular, in the quantum theory of the Doppler effect.[6,7]

The question of the validity of (1) arose in connection with the still continuing discussion of the energy and momentum tensor in a medium. Whereas according to Minkowski the relation between the momentum density and the energy should be taken to be (1), according to Abraham

$$\mathrm{p}_A = E/nc, \tag{2}$$

where E is the energy density. The correctness of Abraham's argumentation, which was particularly convincingly verified recently by Skobel'tsyn,[8,10] does not affect the validity of (1) in the considered radiation problems. In fact, in these problems it is necessary to know the recoil that the radiator acquires when it emits or absorbs a photon. Obviously this quantity is of the same nature as light pressure. Yet for light pressure we must assume relation (1) in either the Minkowski or the Abraham approach.

A photon in a medium is obviously not a free particle. The wave propagation is obtained as a result of coherent addition of waves of individual atoms. Thus, of importance for the onset of the wave is the collective motion that occurs in the atoms of the medium. This is a characteristic property not of a particle but of a quasiparticle (for example, in analogy with phonons).

From this point of view, we must, in analogy with quasiparticles, ascribe to the photon in a medium, a velocity

$$\mathbf{w} = d\omega/dk \tag{3}$$

which is equal to the group velocity of light. We shall consider the case of negative **w**. As seen from (1) and (3), the photon momentum **p** and its transport velocity should in this case be oppositely directed (this means formally that the mass of the quasiparticle is negative[1]).

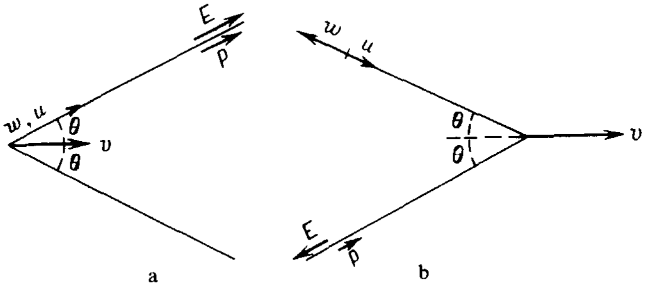

FIG. 1.

We turn now to the Cerenkov effect in a medium with negative group velocity,[7,12] as represented in Fig. 1 (which duplicates, with some additions, Fig. 2 of Ref. 7). In the usual case of positive group velocity (we assume the medium to be optically isotropic) the group velocity **w** of the radiated waves, as well as the phase velocity **u**, is directed at a characteristic acute angle with $\cos\theta = c/vn$ to the particle velocity (Fig. 1a). Obviously, the radiated energy and the momentum **p** of the quasiphoton propagate in the same direction. The momentum carried away slows down the particle.[2,13] Starting out with this picture in the case of negative group velocity, we obtain an unexpected result. The phase and group velocities are in this case oppositely directed, the energy should depart, and consequently **w** should be directed away from the particle. This leads to the picture[7,12] of Fig. 1b, in which **w** makes an angle $\pi - \theta$ with the velocity, and the phase velocity **u**, as before, makes an angle θ, so that the wave travels towards the particle. To make deceleration possible, the momentum should be carried away in the same direction as **u**, and the velocity of the quasiphoton that carries away the momentum should obviously coincide with **w**. In other words, the quasiphoton must in fact be ascribed a negative mass.

A similar unexpected result is obtained also for the refraction of light by a medium with a negative group velocity. (Refraction of light in a medium with $\mathbf{w} < 0$ was considered by Mandel'shtam.)[14] Figures 2a and 2b show the cases of refraction of light in a medium for $\mathbf{w} > 0$ and $\mathbf{w} < 0$, respectively. It is known that an analogy exists here with the Cerenkov effect (Ref. 4),[2] and it can be shown that Fig. 1 and Fig. 2 are

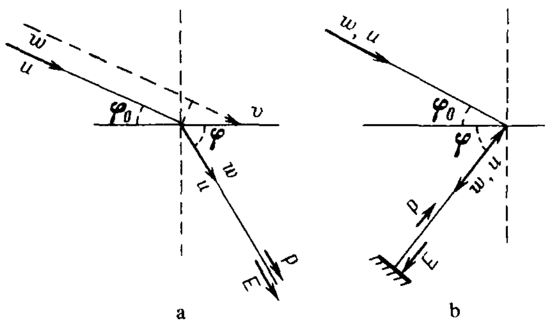

FIG. 2.

consequences of each other. The analogy consists in the following: the propagation velocity of the front of the wave incident on the vacuum along the interface is $v = c/\cos\phi_0$, where ϕ_0 is the angle between the vector **k** and the boundary of the medium. The angle ϕ between the refracted wave and the surface of the medium is obviously given then by $\cos\phi = c/nv$.

Since refraction of the light changes only that vector component $\mathbf{k}_z$ normal to the interface, it can be assumed that the light incident on this interface will act on the surface of the medium with a force that is only normal to the surface. The absence of a force component directed along the surface leads to the requirement that the momentum of the photon of the medium differ from the momentum in vacuum likewise by the cosine of the angles with the surface, i.e., $\cos\phi_0/\cos\phi = n$, which in fact corresponds to Eq. (1).3)

We consider now the refraction of light by the surface of a medium in which the group velocity is negative. This case is shown in Fig. 2b. The group velocity is directed in this case from the boundary into the interior of the medium, while the phase velocity, on the contrary, is directed from the medium to the surface. Both the incident and the reflected ray are on the same side of the interface.[14] The momentum of the quasiphoton should be directed along the phase velocity. This is necessary if the light pressure on the surface is to have in this case, too, only a normal component. On the other hand, the light carries energy from the surface into the interior of the medium, and obviously momentum can be carried only with the energy. Here again we find, just as in the Cerenkov effect, that the velocity of the quasiphoton and the direction of its momentum are of opposite sign. Let us imagine now a black surface inside the medium with negative group velocity (Fig. 2b). It should receive the light momentum directed from the surface. Thus, the light pressure inside the medium with negative velocity should also be negative.

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1) The question of the photon mass in an ordinary medium ($\mathbf{w} > 0$) is considered in Ref. 11. In a medium without dispersion $\mathbf{w} = c/n$, so that putting $\mathbf{p} = m'\mathbf{w}$ we obtain $m' = (n^2/c^2)\hbar\omega$.

2) The terminology is discussed in a paper by Bolotovskiĭ and Ginzburg.[15]

3) This argument is not new (see Refs. 16 and 11).

[1] I.M. Frank, Soobshch. JINR R4-95899, Dubna, 1976.

[2] V.L. Ginzburg, Zh. Eksp. Teor. Fiz. **10**, 584 (1940).

[3] I.M. Frank, Usp. Fiz. Nauk **30**, 149 (1946).

[4] I.M. Frank, Izv. Akad. Nauk SSSR Ser. Fiz. **6**, 3 (1942).

[5] V.L. Ginzburg and V.Ya. Eidman, Zh. Eksp. Teor. Fiz. **36**, 1823 (1959) [Sov. Phys. JETP **9**, 1300 (1959)].

[6] V.L. Ginzburg and I.M. Frank, Dokl. Akad. Nauk SSSR **56**, 583 (1974).

[7] I.M. Frank, Nobel Prize Lecture (Science, Vol. 131, 1960).

[8] D.V. Skobel'tsyn, Preprint Phys. Inst. edited by P.N. Lebedev, No. 160, 1972; Usp. Fiz. Nauk **110**, 253 (1973) [Sov. Phys. Usp. **16**, 381 (1973)].

[9] D.V. Skobel'tsyn, C.R. Acad. Ec. Paris **280**, Ser. B **251**, 287 (1975).

[10] D.V. Skobel'tsyn, Usp. Fiz. Nauk **122**, 295 (1977) [Sov. Phys. Usp. **20**, 528 (1977)].

[11] F.R. Tangherlini, Phys. Rev. A **12**, 139 (1975).

[12] V.E. Pafomov, Zh. Eksp. Teor. Fiz. **36**, 1853 (1959) [Sov. Phys. JETP **10**, 1321 (1959)].

[13] I.E. Tamm and I.M. Frank, Tr. Fiz. Inst. Akad. Nauk SSSR **2**, 63 (1944).

[14] L.I. Mandel'shtam, Sobranie Trudov (Collected Works) **5**, 465 (1950).

[15] B.M. Bolotovskiĭ and V.L. Ginzburg, Usp. Fiz. Nauk **106**, 577 (1972) [Sov. Phys. Usp. **15**, 184 (1972)].

[16] G. Györgyi, J. Phys. **28**, 85 (1960).