# The character of a $\pi$-condensate phase transition at finite temperature

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The results of analytic calculations of the critical temperature of pion condensation are presented. It follows from these results, in particular, that at sufficiently high temperature the pion condensation proceeds like a first-order phase transition.

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Interest in the study of the properties of nuclear matter at high density and temperature has greatly increased in recent years. Particular attention is being paid to the problem, first investigated by Migdal, of pion condensation and of the possible existence of anomalous nuclear states.[1–4]

To observe manifestations of $\pi$-condensate phase transitions in experiments on collisions of heavy ions of high energy, it is necessary to know the temperature dependence of such characteristics as the critical density, energy, etc. Similar problems arise also in the study of pion condensation in neutron stars, during the initial state of the evolution, when the temperatures reach several dozen MeV.

To study pion condensation at finite temperatures it is convenient to use the approach developed[2,5,6] for $T=0$. In Refs. 5 and 6 they calculated the energies of the nucleon and $N^*(1232)$ quasiparticle at arbitrary amplitude $a$ of a charged condensate field in the form of a traveling wave: $a=F_\pi \times \tan\theta$ ($F_\pi = 189$ MeV is the pion decay constant, $\theta$ is the angle of the chiral rotation: $0 \leqslant \theta \leqslant \pi/2$) in a sufficiently realistic model that takes into account the $S$- and $P$-wave $\pi NN$ and $\pi NN^*$ interactions, the $\pi\pi$ interaction, and the nucleon correlations.

If a second-order transition takes place, then near the critical point ($\theta \to 0$) the increments of the free energy $F$ or of the thermodynamic potential $\Omega$ are connected with the polarization operator $\Pi$ of the pion in the nucleon medium, expressed in terms of the corresponding variables ($\hbar = c = m_\pi = 1$):

$$\delta F(n, T) = \delta\Omega(\mu, T) = (1 + k^2 + \Pi - \omega^2)\,\frac{F_\pi^2\,\theta}{8}\,. \qquad (1)$$

The chemical potential $\mu$ and the baryon density are connected by the relation $n = -\partial\Omega/\partial\mu$, and $\omega$ and $k$ are the frequency and wave number of the condensate field.

Knowing the dependence of $\Pi$ on $T$ and using the same methods as in Ref. 2, we can obtain the dependence of the critical density $n_c(T)$, of the pion condensation on the temperature, or its inverse $T_c(n)$. On the $n$-$T$ plane the states with pion condensate lie below the $T_c(n)$ curve.

In the calculation of $T_c(n)$ we neglect the temperature dependence of the local characteristics of the system, such as $f$, $g'$, $m$, and others (see below). Owing to the strong dependence of the principal terms of $\Pi$ on the temperature, we can separate two characteristic regions, I and II.

Region I: $T \ll \Delta \equiv m_{N^*} - m_N$. In this region of $T$ only the spin–isospin nucleon states are filled. All the $N^*$-particle states are separated by a gap $\Delta$, and in view of the presence of the factor $e^{-\Delta/T}$ their occupation is exponentially weak. The region $T \ll \Delta$ breaks up into two subregions: $T \ll \epsilon_F$ (Ia) and $\epsilon_F \ll T \ll \Delta$ (Ib).

For the pole term $\Pi_p$ corresponding to a diagram of the particle–hole type, the temperature $T \ll \epsilon_F$ is low, and $T \gg \epsilon_F$ is high. For the resonant term $\Pi_R$, which is connected with the $N^*$ particle–nucleon hole graphs, the temperature $T \ll \Delta$ is low and the temperature dependence of this term is due only to allowance for recoil. For the $S$-wave term $\Pi_S$, the temperature $T \ll \epsilon_F$ is low. In this limit $\Pi_S \sim (n^n - n^p)$ and does not depend on the temperature. At $T \gg \epsilon_F$ we have $\Pi_s \to 0$ ($\Pi_S = 0$ in an isosymmetrical medium).

In region I we have

$$\Pi = -f^2 k^2 \Gamma A(k, \omega, T) + \Pi_S\,, \quad \Gamma = (1 + g' A)^{-1},$$
$$A(k, \omega, T) = \frac{2m}{\pi^2}\{p_F^n\,\tilde{\Phi}^n(k, \omega, T) + p_F^p\,\tilde{\Phi}^p(k, -\omega, T)\}\,. \qquad (2)$$

The indices $n$ and $p$ stand here for neutrons and protons, $f = 1.0$, $m$ is the effective mass of the nucleon, ($m \approx m_N$), $p_F^{n,p} = (3\pi^2 n^{n,p})^{1/3}$ is the Fermi momentum,

$$\tilde{\Phi}(k, \omega, T) = \Phi_1(k, \omega, T) + \frac{32}{25}\,\Phi_1(k, \omega - \Delta, T)\,, \qquad (3)$$

The factor $\Gamma$ takes into account the nucleon correlations and $g'$ is the Fermi-liquid parameter.[2] To simplify the analytic expressions, we assume the $NN$, $NN^*$ and $N^*N^*$ correlations to be identical.

The function $\Phi_1(k,\omega,T)$ can be expanded in the form:

Ia:

$$\Phi_1(k,\omega,T)=\Phi_1(k,\omega)\left\{1-\frac{\pi^2}{12}\left[1-\left(\frac{\omega}{kv_F}\quad\frac{k}{2p_F}\right)^2\right]^{-1}\left(\frac{T}{\epsilon_F}\right)^2\right\},\tag{4}$$

where $v_F=p_F/m$, and $\Phi_1(k,\omega)\equiv\Phi_1(k,\omega,T=0)$ is the same as in Ref. 2.

Since in a neutron star the temperatures $T\sim 0.1$, which are of physical interest, pertain to region Ia, we present for the other regions only the result for a medium with $N=Z$, when $\omega=0$.

Ib:

$$\tilde{\Phi}(k,0,T)=\frac{\epsilon_F}{3T}\left\{1-\frac{k^2}{12mT}+\frac{64}{25}\left(\frac{T}{\Delta}\right)\left[1+\frac{\pi^2k^2}{3P_F^2}\left(\frac{T}{\Delta}\right)^2\right]\right\}.\tag{5}$$

On the basis of the foregoing expressions we can draw the following conclusions. In region Ia in a medium with $N=Z$ we have $T_c(n)\sim\epsilon_F[(n-n_c)/n_c]^{1/2}$, when $n_c=n_c(T=0)$, and the critical momentum $k_c$ of the condensate field has a positive increment $\propto T^2$. In region Ib, the function $T_c(n)$ depends strongly on the choice of the parameters. In a neutron medium in region I, the $T_c(n)$ plot takes the opposite course ($\partial T_c/\partial n<0$), i.e., raising the temperature contributes to the development of the condensate. The reason is that in this case $\omega\sim(\Delta-\omega)\gg k^2/2m$ and, as seen from (4) both the pole and the resonant terms of $\tilde{\Phi}$ have positive temperature increments. In a medium with $N=Z$ ($\omega=0$) these increments have opposite signs. Generally speaking, when the parameters of the theory are varied in reasonable limits, the inverse behavior of the $T_c(n)$ curve in region I is possible also in symmetrical nuclear matter. This possibility becomes even more likely if the assumption[1,2] that there are only $NN$ correlations, while the $NN^*$ and $N^*N^*$ correlations are equal to zero, is valid. On the other hand, if the situation is reversed, i.e., the isobar role is small, then in the region Ib we have $T_c(n)\approx f^2(1-\gamma)n$ and $k_c^2=[12mT/(1-\gamma)']^{1/2}$, where $\gamma=g'/f^2$.

Region II: $T\gg\Delta$. In this region, all 20 spin–isospin states, of four nucleons and 16 $N^*$ particles, are filled simultaneously. At $T\gtrsim\Delta$ it is necessary to consider also diagrams of the isobar–isobar hole type, which behave in analogy with the diagrams of the nucleon particle–hole type. The high-temperature limit for $\Pi_R$ sets in only at temperatures $T\gg T^*=m\Delta^2(1-\gamma)/3$ [see formula (7)]. In this region the polarization operator can be represented in the form

$$\Pi=-\frac{cnf^2k^2}{T}\tilde{\Gamma}\left(1-\frac{k^2}{12mT}\tilde{\Gamma}\right)+f^2(c-1)mn\tilde{\Gamma}^2\left(\frac{\Delta}{T}\right)^2,\tag{6}$$

$$\tilde{\Gamma}=(1+g'cn/T)^{-1};\; c=189/125\cdot$$

The critical parameters in region II are given by

$$n_c(T) \approx \frac{T}{cf^2(1-\gamma)}\left[1+\frac{1}{\sqrt{3mT(1-\gamma)}}+O\left(\frac{1}{T}\right)\right], k_c^2 \approx \sqrt{\frac{12mT}{1-\gamma}}\left(1+\frac{T^*}{T}\right). \quad (7)$$

In Ref. 7, in the numerical calculation of $T_c(n)$, the $T$-dependence of the resonant term $\Pi_R$ was discarded, and the occupation of the $N^*$-particle states at high temperature was not taken into account, the result therefore differs substantially.

The free energy of the system at finite temperature can be analytically calculated also in the case of a limiting condensate field ($\theta = \pi/2$); this makes it possible to check on the possibility of a first-order phase transition. In the limiting field at $T = 0$, one Fermi sphere of quasiparticles, which are superpositions of nucleons and $N^*$ particles, is filled.[5] With increasing temperature, excited states separated from the ground state by gaps $\Delta/3$, $2\Delta/3$, and $\Delta$ are gradually filled. At $T \ll \Delta/3$ the free-energy increment connected with the appearance of the limiting condensate field is proportional to the density $n$, and is therefore independent of the temperature. The corresponding critical density, starting with which this state becomes energywise favored, is calculated in Refs. 5 and 6 [$n_c(\theta = \pi/2) < n_c(\theta \to 0)$ at $T \approx 0$]. At $T \gg \Delta$ a temperature dependence appears, and the expression for the critical parameters takes the form

$$n_c(T;\theta = \pi/2) \approx \frac{T}{cf^2(1-\gamma)}\left[1+O\left(\frac{1}{T}\right)\right], k_c^2 \approx \frac{4T}{fF_\pi}\sqrt{\frac{3}{c(1-\gamma)}}\ .$$

It is seen that the expression for $n_c(\theta = \pi/2)$ agrees, accurate to a small correction $O(1/T^{1/2})$, with expression (7) for $n_c(\theta \to 0)$. However, when this correction is taken into account, at temperatures $T \gg \Delta$ the inequality $n_c(\theta = \pi/2) < n_c(\theta \to 0)$ is satisfied, i.e., with increasing density the state with the limiting field becomes energywise favored earlier than the state with the weak field. Thus, at sufficiently high temperatures $T \sim 1$ or even earlier, a first-order phase transtion takes place. This circumstance can greatly influence the dynamics of the collision of heavy ions.


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