Soliton interaction in a liquid with gas bubbles

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The interaction of two solitons moving towards each other or following each other was investigated experimentally in a liquid with gas bubbles. It is shown that when the solitons collide they interact like linear waves, and in the case when one soliton overtakes the other different forms of the nonlinear interaction are observed.

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It was shown in Refs. 1 and 2, on the basis of numerical and physical experiments, that an initial pressure perturbation in a liquid containing gas bubbles evolves in accordance with the Burgers–Korteveg–de Vries (BKdV) equation

$$P_\tau + PP_\xi - \mathrm{Re}^{-1} P_{\xi\xi} + \sigma^{-2} P_{\xi\xi\xi} = 0. \tag{1}$$

At certain parameters of the perturbation (amplitude, width) and of the medium (bubble radius R_0, gas volume content ϕ_0, thermophysical properties of the gas in the bubbles), such that $\sigma/\mathrm{Re} \ll 1$ and $\sigma \sim \sigma_c$, a solitary wave—soliton—is produced in the medium and its shape is determined by the relation

$$\Delta P(x)/\Delta P = \mathrm{sech}^2(x/\delta), \tag{2}$$

where ΔP is the amplitude of the pressure perturbation, δ is the half-width of the soliton, and x is the longitudinal coordinate.

This evolution of the process is in full agreement with the solution of the BKdV equation, which goes over into (1) at $\mathrm{Re}^{-1} = 0$.

Recently solitons have been actively investigated in a number of branches of physics, as applied to media in which nonlinearity and dispersion effects occur.[3-5] This interest is due to a number of remarkable properties of solitons, which manifest themselves, for example, when they interact. On the other hand, these processes make it possible to reveal the nonlinear, dispersion, and dissipative properties of the medium.

To study the character of the interaction of solitons in a liquid with gas (CO_2) bubbles at $\sigma/\mathrm{Re} \ll 1$, experiments were performed with a setup of the "shock tube" type.[2]

Two solitons were formed in a medium by a successive time-synchronized breaking of diaphragms of two high-pressure chambers, located one behind the other in the upper part of the tube. By choosing the parameters of the solitions and of the medium, it was possible to observe the co-moving interaction of these solitons at a distance 1–2 meters from the entry of the pulses into the medium, and also opposing interaction between the second soliton with the first when the latter is reflected from the lower end of the low-pressure chamber.

Piezoelectric pickups were placed along the working section at a distance L from the entry of the signal into the medium.

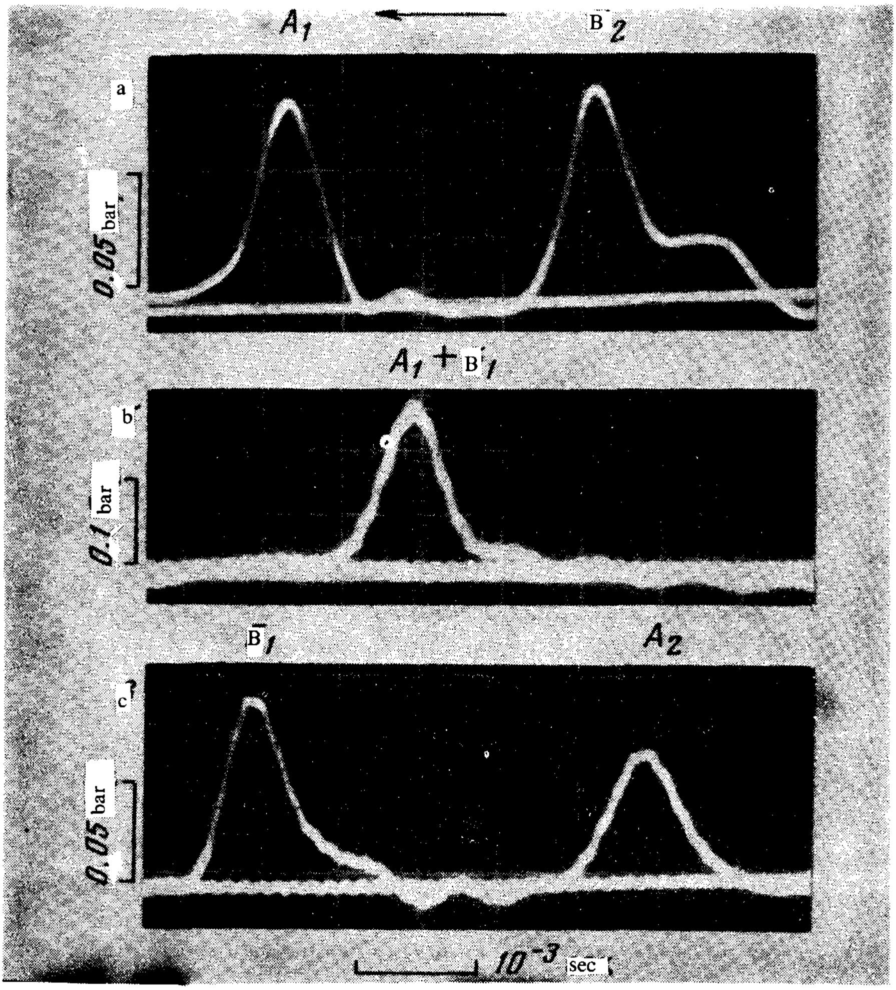

FIG. 1. Opposing interaction of two solitons a—$L = 0.7$ m, b—$L = 0.91$ m, $\phi_0 = 1.4\times10^{-2}$, $R_0 = 1.15\times10^{-3}$m, c—$L = 1.11$ m.

The process of the opposing interaction is illustrated in Fig. 1, which shows the stages of complete coalescence of the initial pulses A_1 and B_1 Fig. 1(a) into a single pulse Fig. 1(b), as well as their subsequent separation Fig. 1(c). As a result of interaction of two solitons with respective amplitudes ΔP_1 [A in Fig. 1(a)] and ΔP_2 [B in Fig. 1(a)], a certain perturbation with amplitude ΔP is produced Fig. 1(b), having a shape that does not agree with that of the soliton (2). As seen from Fig. 2, at the instant when the solitons are superimposed, their amplitudes add up in accord with the behavior of linear waves, and the results of the experiments are described by the relation $\Delta P_c = \Delta P_1 + \Delta P_2$ (line 3 in Fig. 2). The only result of the nonlinear interaction, as is well known,[4,5] is a phase shift of the perturbations. However, for small-amplitude waves this shift did not exceed the accuracy of our measurements, so that we were unable to obtain quantitative results on the phase shift. The character of the interac-

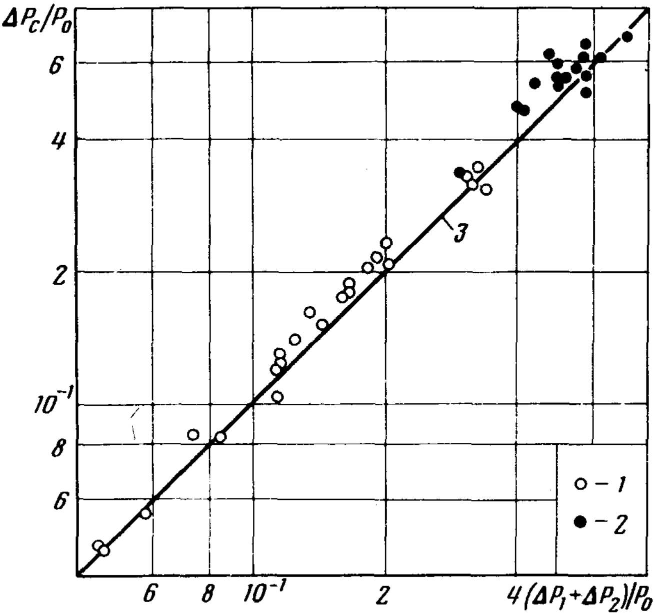

FIG. 2. Pressure amplitude at the instant of coalescence of two solitons: $L = 0.7$ m, 1—$R_0 = 1.15\times 10^{-3}$ m, $\phi_0 = 1.4\times 10^{-2}$ m, 2—$R_0 = 1.4\times 10^{-3}$ m, $\phi_0 = 0.9\times 10^{-2}$.

tion described above agrees with the numerical solutions of Boussinesq and with the experiments in Ref. 5.

The co-moving interaction, wherein a soliton with a larger amplitude overtakes a soliton with a smaller amplitude, is characterized by a nonlinear interaction between them. For perturbations with nearly equal amplitudes, no coalescence takes place, just as in the case of frontal collision. The perturbations approach each other until their amplitudes become equal [Fig. 3(a)]; subsequently the amplitudes of the front soliton

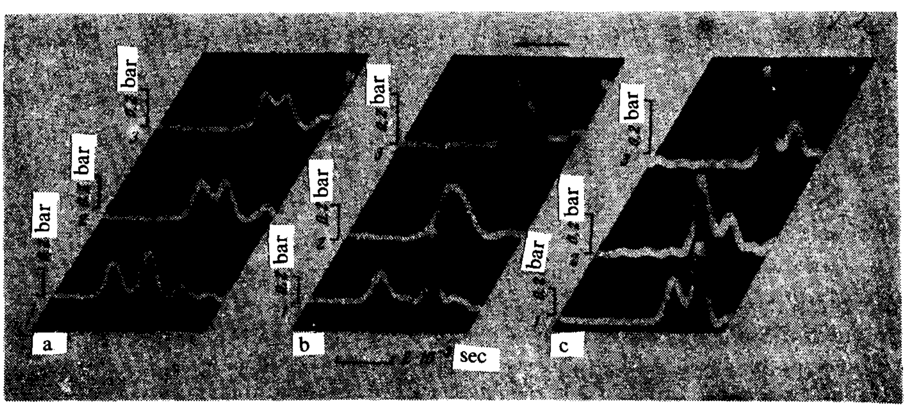

FIG. 3. Interactions of solitons when one soliton takes over the other: for $L = 0.14$ m, $R_0 = 1.4\times 10^{-3}$ m, $\phi_0 = 1.4\times 10^{-2}$:a—$\Delta P_2/\Delta P_1 = 1.3$; 1—$L = 0.14$ m, 2—1.05 m, 3—1.3 m, b—$\Delta P_2/\Delta P_1 = 2.5$; 1—$L = 0.14$ m, 2—1.55 m, 3—1.8 m, c—$\Delta P_2/\Delta P_1 = 1.85$; 1—$L = 0.14$ m, 2—1.55 m, 3—1.8 m.

(initially smaller) increases and the perturbations move apart.

With increasing ratio of the amplitudes, the minimal distance of approach of the solitons decreases, and at a value $\Delta P_2/\Delta P_1 \approx 2.5$ an instant sets in when they practically coalesce [Fig. 3(b)]. This value differs somewhat from the value 2.62 predicted theoretically in Ref. 7, because it is difficult in experiment to determine exactly the instant of complete coalescence of the solitons. With further increase of the amplitude ratio, the solitons interact like linear waves: the larger soliton overtakes the smaller one, they coalesce, and then break up into two solitons which have, with damping taken into account, their initial amplitudes. We note that in experiments, at all amplitudes, the interaction resulted in decay into two solitions, or signs of it were noted but it did not have time to take place because of the limited length of the working section [Fig. 3(c)].

The types of soliton interactions considered in the paper were also investigated experimentally before in electromagnetic systems[4] and in an ion plasma.[6]

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