

# Magnetic ordering in superconductors

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Magnetic ordering in superconductors is discussed in the self-consistent field model with several coupled parameters.

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A number of ternary compounds with transition metals have been synthesized recently. Despite the high concentration of the rare-earth ions which are periodically dispersed in the lattice, these compounds have high superconducting-transition temperatures  $T_c$  and exhibit properties connected with magnetic ordering of the rare-earth ions without<sup>1</sup> and with<sup>2</sup> destruction of the superconducting state.

When atoms having localized magnetic moments are introduced in a superconductor, a disordered state is produced and  $T_c$  is lowered by an amount on the order of the Curie temperature  $\Theta$  as a result of the exchange interaction of the conduction electrons with the spin of the ion.<sup>3</sup> Magnetic ordering introduces an additional mechanism that suppresses the superconductivity. In the case of ferromagnetic ordering, exchange magnetization of the conduction electrons occurs even in first order in the exchange-interaction constant  $J$  and leads to an electron Fermi surface with opposing spin orientations in the effective magnetic field  $H_{\text{eff}} \sim xJ$ , where  $x$  is the concentration of the magnetic ions, and superconducting pairing is possible only at concentrations on the order of 1%.<sup>4</sup> In antiferromagnetic ordering there is no exchange magnetization effect, but the appearance of a magnetic superlattice leads to a change in the energy spectrum of the conduction electrons and to formation of an energy gap, characterized by  $J$ , in directions close to the reciprocal-superlattice vector.

In compounds with transition metals, in the absence of exchange interaction of the  $d$  electrons with the spin of the ion, the superconductivity is preserved at high ion concentration at which superconductivity in an individual  $s$  band would certainly be impossible. In the "dirty" limit, when a two-band superconductor behaves like a single-band one, a gain by a factor  $(\tau_{sd} + \tau_{ds})/\tau_{sd}$  takes place in the critical concentration of the ions over that in a single  $s$  band, while  $H_{\text{eff}}$  decreases by the same factor. Here  $\tau_{sd}$  and  $\tau_{ds}$  are the path times connected with magnetic scattering in the  $s$  band with transition to the  $d$  band and vice versa.

We shall analyze the interaction of the magnetic ordering with the superconductivity within the framework of the self-consistent-field model. In this case the system is described by two coupled order parameters—the magnetization  $M$  and the superconducting pairing parameter  $\Delta$ . Assuming  $\Theta$  and  $T_c$  to be close to each other, the free energy can be expanded in powers of the order parameter

$$F(T, h) = \int d^3r \left[ \frac{c}{2} (\nabla M)^2 + \frac{a(T)}{2} M^2 + \frac{b}{4} M^4 + \frac{(h - 2\pi M)^2}{8\pi} \right]$$

$$+ \alpha(T) |\Delta|^2 + \frac{\beta}{2} |\Delta|^4 + \frac{\gamma}{2} \left| \left( -i \nabla - \frac{2e}{cA} \right) \Delta \right|^2 + GM^2 |\Delta|^2 \Big]. \quad (1)$$

All the terms but the last have the same meaning as in the corresponding theories with a single order parameter.<sup>6,7</sup>  $a(T)$  and  $\alpha(T)$  vanish at the temperatures  $\Theta$  and  $T_c$ , respectively. The last term in (1) describes the appearance of an energy gap on the Fermi surface in the case of exchange magnetization.<sup>1)</sup> Other examples of free energies with interaction of the order parameter and with the influence of the fluctuations are cited in the review by Patashinskiĭ and Pokrovskiĭ.<sup>5</sup>

In the homogeneous situation when  $M$  and  $\Delta$  are constant, there is no coexistence of the magnetic and superconducting phases in an infinite plate of thickness  $d$  with the easy-magnetization axis parallel to the planes of the plate at

$$g = 2^{1/2} G (\beta b)^{-1/2} > 1 \quad (2)$$

At  $\Theta > T_c$  only the magnetic phase exists, and at  $\Theta < T_c$  a first-order phase transition takes place and the superconductivity is destroyed at the temperature  $T_0$  at which the condensation energies of the phases become equal.

In order of magnitude we have  $g = J_{\text{eff}}^2 / T_c (\Theta \epsilon_F)^{1/2}$  and there are no coexisting phases for a transition metal, where the superconducting pairing and the magnetic ordering are determined by the electrons of one band. In a transition metal in which the  $d$  electrons are not magnetized it is possible to have  $g < 1 (J_{\text{eff}}) = J \tau_{sd} / (\tau_{sd} + \tau_{ds})$  also at high concentrations of the rare-earth ions. In this case it is necessary to take into account the contribution of the diamagnetic term to the interaction of the order parameters.

At  $T_c > \Theta$  the maximum thickness at which the  $SF$  phase is possible is determined from

$$2^{3/2} \pi b^{-1/2} \beta^{1/2} |\alpha|^{-1} [1 - 2\epsilon^{-1} \text{sh} \epsilon (\text{ch} \epsilon + 1)^{-1}] = 1 - g. \quad (3)$$

At large thicknesses we have a first-order transition with destruction of the superconductivity at  $T = T_0$ . Here  $\epsilon = d/\lambda$  and  $\lambda$  is the depth of penetration at  $\Delta^2 = |\alpha|/\beta$ . In thick samples the magnetic moment is screened by superconducting currents and the magnetic field in the sample is zero, just as in the Meissner effect in an external magnetic field. The order of the  $SF$ - $F$  phase transition depends on the thickness and at

$$\frac{d^2}{\lambda^2} < 5 \frac{V}{V - G} \left( 1 - \frac{2V^2}{bB} \right); \quad V = \frac{\pi}{6} \frac{\epsilon^2 \beta}{|\alpha|} + G \quad (4)$$

we have a second order transition. The transition line is determined from

$$16\pi^2 \frac{|q|}{b} = H_c^2(T) \left[ \frac{\epsilon^2}{24} + \frac{G}{4\pi} \frac{|\alpha|}{\beta} \right]^{-1}. \quad (5)$$

At  $\Theta \gg T_c$  and in the absence of exchange interaction ( $G = 0$ ) equations (4) and (5) go over into the conditions obtained in Ref. 8. In thick samples, a first-order transition takes place at

$$4\pi \frac{|\alpha|}{b} = H_c^2(\Delta^2), \quad \Delta^2 = \frac{|\alpha|}{\beta} - G \frac{|\alpha|}{b}. \quad (6)$$

It should be noted here that in type-II superconductors vortical states will exist against the background of the magnetization and the  $SF-F$  transition will be of second order.

For an easy-axis antiferromagnet with an anisotropy constant  $\gamma$  and with order-parameter interaction due to the suppression of the magnetic sublattice we have

$$V_1 l_z^2 |\Delta|^2 + V_2 (l_x^2 + l_y^2) |\Delta|^2, \quad (7)$$

where  $\mathbf{l} = \mathbf{m}_1 - \mathbf{m}_2$  and  $\mathbf{m}_1$  and  $\mathbf{m}_2$  are the sublattice magnetizations. The phase diagram at  $V_2 + \gamma\beta/|\alpha| > (\beta b)^{1/2}$  is shown in Fig. 1, and for the reverse inequality in Fig.

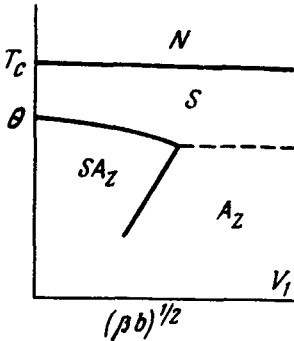


FIG. 1.

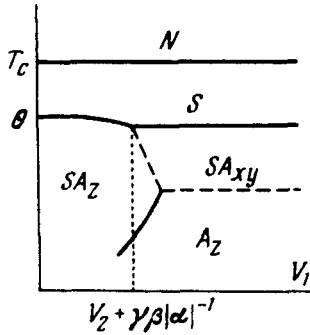


FIG. 2.

2. The solid and dashed lines show second and first order transitions, respectively.

In non-transition metals,  $V(\beta b)^{-1/2} \sim \Theta/T_c$  and the coexistence of the phases is possible at  $\Theta$  much lower than  $T_{c0}$ .

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<sup>1</sup>The usual magnetization  $\mu_0 \sigma h$  can be neglected.

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