Applicability of the Fermat principle to the optics of ultracold neutrons

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We consider the applicability of the Fermat principle to optics of ultracold neutrons (UCN) when the presence of gravitation bends significantly the neutron trajectories. It is shown that the Fermat principle is fully applicable also in the case of neutron waves.

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The ability of ultracold neutrons (UCN) to be reflected from a mirror at any incidence angle gives grounds for hoping that there exists a possibility of producing neutron-optical instruments, and prospectively a neutron mircoscope.[1] Since the UCN wavelength is comparable with the ultraviolet wavelength, an important role in the formation of the image is played not only by the geometrical-optics law, but also by the features typical of wave optics. The role that is played here by the wave properties can be seen, for example, from the diffraction and interference experiments of Schechehofer and Steyerl.[2] They already used in fact a rather complicated optical system, in which the neutron wave underwent a large number of reflections from flat mirrors.

It is known that a very significant feature of UCN is the change of the neutron velocity with changing height. The trajectory is then bent in a field of gravitational forces. This effect is appreciable: the UCN cannot be raised to more than 1–2 m above their source. The influence of the gravitation causes obviously the position of the focus into which a neutron beam is gathered to depend on neutron initial velocity (chromatic aberration), and this influence is appreciable even for neutrons with velocity close to the maximum value. It is this which makes the optics of ultracold neutrons peculiar.

In ordinary optics, the rays gathered into the focus of a point source should have identical phases. This is a consequence of the Fermat principle, according to which the path of the ray is such that its optical length is minimal. In the ideal case, when light is focused into a point, this minimum condition should be satisfied for each ray from a continuous sequence of rays arriving at the focus, and this obviously is possible only if the optical length is the same. Equality of the phases of the waves at the focus is a substantial characteristic of the image. Since the path length to the focus differs for different neutron trajectories, and the wave vector k varies along the trajectory, this raises the question of the applicability of the Fermat principal to the optics of UCN.

The problem of wave propagation in a potential field can be described by introducing a refractive index. In the case of a potential determined by gravitation, we obtain from the energy conservation law

$$k_1^2 = k^2 - \frac{2m}{\hbar^2} U, \qquad U = mgz. \tag{1}$$

Here U is the change of the potential after a rise to a height z above a certain level arbitrarily taken to be zero height, and $k^2 = m^2v^2/\hbar^2$, where v is the neutron velocity at $z = 0$.

The square of the refractive index is therefore

$$n^2(v, t) = \frac{k_1^2}{k^2} = 1 - \frac{2gz}{v^2}. \tag{2}$$

This equation allows us to reduce the problem of propagation of neutron waves to the optial problem of propagation of light in a medium with a refractive index that is a function of z. Since the Fermat principal does not call for the assumption that the medium be optically homogeneous, it is consequently applicable also in the case of neutron waves.

[1]I.M. Frank, Priroda No. 9, 24 (1972).
[2]H. Schechehofer and A. Steyerl, Phys. Rev. Lett. **39**, 1310 (1977).