We then calculate within the framework of the WKB method the average distance r between the electron and the center. It turns out that it is important to take into account not only the classically accessible region, but also the inaccessible one. For the case $Z = Z_{cr} = 170$, the value of r turned out to be 0.24 (as against 0.29 from the exact calculation [3]), i.e., larger than the width of the classically accessible region r_0 (see Fig. 1), which equals 0.20 at $Z_{cr} = 170$. This means that in the region $Z = Z_{cr}$ the electron in the $1s_{1/2}$ state spends the greater part of the time in classically inaccessible region.

The agreement between the WKB approximation and the exact calculations for terms with large quantum numbers turned out to be, naturally, even better than for the $1s_{1/2}$ term.

I am grateful to A.B. Migdal for valuable advice.

[1] J. Heading, Introduction to Phase Integral Methods, Methuen, 1962.
[2] A.B. Migdal and V.P. Krainov, Priblizhennye metody kvantovoi mekhaniki (Approximate Methods of Quantum Mechanics), Nauka, 1967.
[3] V.S. Popov, Yad. Fiz. 12, 429 (1970) [Sov. J. Nucl. Phys. 12, 235 (1971)].

COLLECTIVE EFFECTS IN DISCRETELY-ELECTRONIC BEAMS

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The purpose of the present communication is to call attention to collective effects that arise as a result of the discrete character of the elementary charge in beams of electrons (or other charged particles) in a vacuum, and make it possible to liken such beams, in certain respects, to pseudocrystalline structures.

Assume that the electrons move in vacuum along linear trajectories in a strong longitudinal magnetic field. At a given beam current I, the average longitudinal distance between neighboring beam electrons is $a = ev/I$, where $e = |e|$ is the elementary charge and v the velocity of the electrons. If the magnetic field is sufficiently strong, so that the transverse dimension of the beam is $d \ll a$, then such a thin beam has the form of a linear discretely-electronic chain recalling certain known models of one-dimensional crystals.

The initial distribution of the longitudinal coordinates of the electrons in the chain, determined by the shot effect of the cathode, by the thermal velocities of the electrons, and by the conditions for the formation of the thin beam, has a random character. Therefore the interval between any two neighboring electrons is $a_i(t) \neq a$. With further linear motion, a tendency for ordering occurs in the chain under the influence of the longitudinal Coulomb-repulsion forces, i.e., a tendency of the potential energy of the chain to become minimal as $a_i(t > \tau) \to a$, where τ is a certain relaxation time. This tendency is resisted by the longitudinal thermal motion of the electrons, thus imposing on the temperature T of the ordered beam the limitation

$$|z/a| \approx [akT/4{,}8e^2]^{1/2} = 0.46\left[\frac{a}{r_0}\frac{kT}{mc^2}\right]^{1/2} \ll 1.$$

Here z is the longitudinal coordinate of the electron, $z = 0$ corresponds to the equilibrium position of the electron in a perfectly ordered chain at $a_i = a$,

k is Boltzmann's constant, m and r_0 are the mass and classical radius of the electron, c is the speed of light, and the electric field of the linear perfectly ordered chain, acting on one of the electrons which is displaced from the equilibrium position ($z = 0$, $\rho = 0$) to a point with cylindrical coordinates z and ρ, is characterized by the two components

$$E_z = \frac{e}{a^2}\sum_{\nu=1}^{\infty}\nu^{-2}\left\{\left(1-\frac{1}{\nu}\frac{z}{a}\right)\left[\left(1-\frac{1}{\nu}\frac{z}{a}\right)^2+\nu^{-2}\left(\frac{\rho}{a}\right)^2\right]^{-3/2}-\right.$$

$$\left.-\left(1+\frac{1}{\nu}\frac{z}{a}\right)\left[\left(1+\frac{1}{\nu}\frac{z}{a}\right)^2+\nu^{-2}\left(\frac{\rho}{a}\right)^2\right]^{-3/2}\right\}\approx 4{,}8\frac{e}{a^2}\frac{z}{a}\left\{1+1{,}7\left[\left(\frac{z}{a}\right)^2-\frac{3}{2}\left(\frac{\rho}{a}\right)^2\right]\right\}$$

and

$$E_\rho = -\frac{e}{a^2}\frac{\rho}{a}\sum_{\nu=1}^{\infty}\nu^{-3}\left\{\left[\left(1-\frac{1}{\nu}\frac{z}{a}\right)^2+\nu^{-2}\left(\frac{\rho}{a}\right)^2\right]^{-3/2}+\right.$$

$$\left.+\left[\left(1+\frac{1}{\nu}\frac{z}{a}\right)^2+\nu^{-2}\left(\frac{\rho}{a}\right)^2\right]^{-3/2}\right\}\approx -2{,}4\frac{e}{a^2}\frac{\rho}{a}\left\{1+1{,}3\left[4\left(\frac{z}{a}\right)^2-\left(\frac{\rho}{a}\right)^2\right]\right\}$$

and by the potential

$$U(z,\rho)\approx 1{,}2\frac{e}{a}\left\{\left(\frac{\rho}{a}\right)^2\left[1-0{,}65\left(\frac{\rho}{a}\right)^2+5{,}2\left(\frac{z}{a}\right)^2\right]-2\left(\frac{z}{a}\right)^2\left[1+0{,}85\left(\frac{z}{a}\right)^2\right]\right\},$$

which has a saddle-like distribution (the approximate equations are valid in the case of small displacements from equilibrium $|z/a| \ll 1$ and $\rho/a \ll 1$).

The longitudinal magnetic field intensity necessary to produce a thin ($d \ll a$) pseudocrystalline beam can be approximately estimated from the balance of the radial forces (centrifugal, Lorentz, and Coulomb) acting on the electron

$$H \approx 3{,}1[mc^2a^{-3}]^{1/2} = 1{,}7\cdot 10^{10}\left(\frac{a_0}{a}\right)^{3/2}.$$

The transverse component of the electron velocity is set in this case by its potential $U(z, \rho)$, and the formula for the constant a_0 is given below.

So long as the characteristic linear dimension $\Delta z = 2|z|$ (the longitudinal electron localization interval) greatly exceeds the de Broglie wavelength, i.e., so long as $|z| \gg \pi\hbar/mv$ (h is Planck's constant), it is possible to dispense with a consistent quantum mechanical approach to the problem and to use only the uncertainty relation for the lower estimate of the minimum spatial period a of a pseudocrystal, $a \gg a_0 = \hbar^2/2me^2 \approx 0.3$ Å, characteristic of atomic dimensions and of the constants of ordinary crystals. We have used here the obvious relation

$$\Delta z\,\Delta p_z = 2\Delta z\left[-2em\,U\left(\frac{\Delta z}{2}\right)\right]^{1/2}$$

between the interval Δz and the momentum scatter $\Delta p_z = m\Delta v$, which holds for an electron oscillating in the potential field $U(z,\rho)$.

Pseudocrystalline electron beams, unlike true crystalline structures, retain their stability only in motion ($|v| > 0$), and have no electric neutrality. Therefore, besides the collective effects that have a greater or lesser similarity to phenomena in true crystals, particular interest attaches to collective resonant radiative processes occurring when the electronic pseudocrystal interacts as a unit with external fields and with matter.

The resonant properties of the pseudocrystal can be observed, for example, when it interacts with an electromagnetic wave of frequency $\omega_n = 2\pi nI/e$ ($n = 1, 2, \ldots$). In particular, in full accord with the laws of stimulated emission, such a beam is capable of stimulated emission or resonant absorption of photons with energy $E_n = \hbar\omega_n$. The highest attainable order of the overtone $n = n_0$ depends on the degree of ordering of the pseudocrystal and consequently on its temperature T

$$n_0 = \left\langle 0{,}55\left[\frac{r_0}{a}\,\frac{mc^2}{kT}\right]^{1/2}\right\rangle = \left\langle \frac{1300}{\sqrt{T}}\left(\frac{a_0}{a}\right)^{1/2}\right\rangle .$$

The collective radiative processes in electronic pseudocrystals can be observed in different experimental situations: under conditions of the Cerenkov effect, in transition radiation and bremsstrahlung, in the excitation of a thin-layer dielectric resonator (a phenomenon inverse to optical modulation of an electron beam [1]), in synchronous interaction of a diffraction grating with a surface wave or (in the case of hard photons) with spatial harmonics of the field in a single crystal [2], etc. In all these phenomena, as well as in all other radiation processes produced by free electrons, one should expect the appearance of characteristic lines at the resonant frequencies of the pseudocrystal ω_n, with an intensity proportional to the square of the beam current I.

There is undisputed interest also in investigations of the resonant interaction of pseudocrystalline beams with matter possessing quantum transitions with energies $E = \hbar\omega_n$.

It must be emphasized that in direct contrast to the case of modulation of electron beams by extraneous electric fields, say microwave fields, in electronic pseudocrystals the Coulomb forces cause and maintain the ac component of the beam current.

The kinetics of the process of ordering of the pseudocrystal, as manifest both in the cooling of the beam and its thermalization, and an estimate of the relaxation time τ, will be the subject of a separate study.

In conclusion, the table serves as a useful numerical illustration.

I, μA	v/c	a, μ	H, kOe	T, °K	$2\pi c/\omega_1$, μ	n_0
50	0,2	0,192	33	25	0,96	3
200	0,2	0,048	260	100	0,24	3
200	0,2	0,048	260	25	0,24	6
800	0,2	0,012	2100[1)]	100	0,06	6
800	0,2	0,012	2100	25	0,06	12

[1)] Concerning magnetic fields on the order of a megaoersted see, for example, [3].

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[1] H. Schwarz and H. Hora, Appl. Phys. Lett. 15, 349 (1969).
[2] L.A. Rivlin, ZhETF Pis. Red. 1, No. 3, 7 (1965) [JETP Lett. 1, 79 (1965)].
[3] A.M. Andianov, V.F. Demichev, G.A. Eliseev, and P.A. Levit, ibid. 11, 582 (1970) [11, 402 (1970)].

POSSIBILITY OF CONTROLLING THE PARAMETERS OF PHONONLESS LINES WITH THE AID OF ULTRASOUND

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In the case when electrons localized on impurities interact strongly with lattice vibrations, multiphonon processes leading to broad electron-vibrational light-absorption and luminescence bands become probable [1]. If the constant of the coupling with the oscillations is small, then at low temperatures the spectrum constitutes a phononless line with a weakly pronounced vibrational structure. A rise in temperature can lead to a strong release of heat even in the case of weak coupling, owing to stimulated processes in the phonon subsystem.

In the present paper we propose a method for directed variation of the shape of the optical curves by heating the phonon subsystem of the crystal in a narrow spectral region; this heating is produced, for example, by a powerful ultrasonic wave or by stimulated Mandel'shtam-Brillouin scattering of laser radiation. It turns out that this superheating can become strong enough to make it possible to observe ultrasound-stimulated multiphonon absorption of light.

We write the light-absorption coefficient in the form [2]:

$$K(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\Omega t - \Gamma|t|} \langle\langle d^{+}(0)\, d(t)\rangle\rangle\, dt,$$

where d(t) is the operator of the dipole moment in the Heisenberg representation, and the operator of the interaction with the ultrasound is chosen in an approximation that is linear in the phonon operators $b_{\kappa\sigma}$ and $b^{+}_{\kappa\sigma}$,

$$\hat{H}' = \frac{1}{\sqrt{2}} \sum_{i} v_{ii\vec{\kappa}\sigma} \left(b^{+}_{\vec{\kappa}\sigma} + b_{\vec{\kappa}\sigma}\right)$$

with account taken of only the polaron effect; i numbers of electronic states, which are assumed to be nondegenerate; $\vec{\kappa}$ and σ are the wave vector and the polarization index of the ultrasonic wave of frequency ω; Γ is the damping constant of the discrete lines of the optical spectrum. The results of the averaging in (1) depends on the statistical properties of the vibrations introduced in the crystals, and we consider here two limiting cases: 1) an absolutely coherent source and 2) a thermal (Gaussian) source [3]. The calculation procedure is the same as in the problem of calculating $K(\Omega)$ in the presence of electromagnetic radiation of high intensity [3].

For an absolutely coherent ultrasound source we obtain: