of T_1. Thus, the FCRT decrease δ_p and increase the δ_c.

The factors $d(\beta)$ in (4) lead to a dependence of δ_c/δ_p on the angle θ, in quantitative agreement with experiment [3]. This explains also the angular dependence of the effective time T_1 [3] and the dependence of δ_p/δ_c on H [5]. As $f \to 0$ the FCRT effect vanishes, and at sufficiently large f, when $[\langle\nu_{cr}^2\rangle]^{1/2} > \nu_{-1/2m_2m_2}$, the value of $W^{cr}_{Mm_2m_2'}$ does not depend on M, thereby equalizing δ_c and δ_p.

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INSTABILITY OF SPIN WAVES IN FERROELECTRIC ANTIFERROMAGNETS IN EXTERNAL FIELDS

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At the present time we know of a considerable number of ferroelectric magnetically-ordered cyrstals - antiferromagnets and weak ferromagnets (see the review [1]). We shall show that in such crystals it is possible to excite parametrically spin waves by an external electric field. The mechanism of this excitation is close to the known mechanism of parametric excitation of spin waves by an external alternating magnetic field [2], although the two are not equivalent.

Excitation of spin waves is easiest to realize in a crystal under conditions close to critical (i.e., at values of the external magnetic field, temperature, and pressure close to the critical values at which a phase transition takes place in the magnetic system of the crystal). This is due to the fact that in the vicinity of the critical point a relatively small alternating electric field suffices for excitation of the spin waves.

We consider the excitation of spin waves by an alternating electric field in a ferroelectric antiferromagnet in the presence of a constant magnetic field H_0. As an example of the critical point, we analyze the case $H_0 \sim H_c$, where H_c is the field when the magnetic moments of the sublattices turn over. We show that for spin-wave excitation the amplitude of the electric field should exceed a critical value $e_c = e_c^0[1 - (H_0/H_c)^2]^{1/2}$, where $e_c^0 \sim 300$ V/cm.

We start from the known experession for the total macroscopic energy of a magnetically ordered ferroelectric [3] in the absence of a constant external electric field

$$U = \int\left\{\frac{h^2 + \epsilon e^2}{8\pi} - H_0(M_1 + M_2) + F\left(M_1, M_2, \frac{\partial M_1}{\partial x_i}, \frac{\partial M_2}{\partial x_i}, e\right)\right\} dr,$$

$$F = \frac{\alpha}{2}\left(\left(\frac{\partial M_1}{\partial x_i}\right)^2 + \left(\frac{\partial M_2}{\partial x_i}\right)^2\right) + \alpha' \frac{\partial M_1}{\partial x_i}\frac{\partial M_2}{\partial x_i} + \eta M_1 M_2$$

$$- \frac{\beta}{2}\left((M_1 n)^2 + (M_2 n)^2\right) - \beta'(M_1 n)(M_2 n) - \lambda e[M_1 M_2], \qquad (1)$$

where α, α', and η are the exchange-interaction constants, β and β' are the magnetic-anisotropy constants, $\vec{n}$ is a unit vector along the anisotropy axis (z axis), $\vec{e}$ and $\vec{h}$ are the alternating electric and magnetic fields, $\vec{M}_\nu$ (ν = 1, 2) is the density of the magnetic moment connected with ν-th magnetic sublattice, and λ is the magnetoelectric constant, equal to $\sim 10^{-3}$ (according to the estimates of [3]). We have left out from (1) the term responsible for the weak ferromagnetism, since it leads only to the insignificant redetermination of the spectrum of the free oscillations.

The equations of motion of the magnetic moments of the sublattices are (see, e.g., [4])

$$\dot{M}_\nu = g[M_\nu \times H_\nu^{eff}] - \frac{1}{\tau M_0^2}[M_\nu \times [M_\nu \times H_\nu^{eff}]], \qquad (2)$$

where g is the gyromagnetic ratio, τ is the relaxation constant, M_0 is the equilibrium value of the density of the magnetic moment of each of the sublattices and $\vec{H}_\nu^{eff}$ is the effective magnetic field:

$$H_\nu^{eff} = H_0 + h - \frac{\partial F}{\delta M_\nu} + \frac{\partial}{\partial x_i}\frac{\partial F}{\partial(\partial M_\nu/\partial x_i)} .$$

For an antiferromagnet with anisotropy of the easy-axis type, in the case when the external alternating electric field $\vec{e}_p$ and the constant magnetic field $\vec{H}_0$ are directed along the anisotropy axis, Eqs. (2) can be reduced to the form

$$\dot{m}_\pm = \left(\mp igM_0 a - \frac{H_0}{\tau M_0}\right)\ell_\pm + \left(\mp igH_0 - \frac{2\eta}{\tau}\right)m_\pm + g\lambda M_0 e_p m_\pm ,$$

$$\dot{\ell}_\pm = \left(\mp igM_0 2\eta - \frac{H_0}{\tau M_0}\right)m_\pm + \left(\mp igH_0 - \frac{a}{\tau}\right)\ell_\pm - g\lambda M_0 e_p \ell_\pm , \qquad (3)$$

where $m_\pm = m_x \pm im_y$, $\ell_\pm = \ell_x + i\ell_y$, $\vec{m}$ and $\vec{\ell}$ are the spatial Fourier components of the vectors $\vec{M} = \vec{M}_1 + \vec{M}_2$ and $\vec{L} = \vec{M}_1 - \vec{M}_2$, and $a = (\alpha - \alpha')k^2 + \beta - \beta'$, and k is the wave vector of the oscillations (we took into account the fact that η >> 1 and λ << 1; the latter inequality enables us to neglect the internal high-frequency electric field and put $\vec{e} = \vec{e}_p$).

Being interested in parametric excitation of spin waves (see, e.g., [5]), we put $\vec{e}_p = \vec{e}_0 \cos 2\Omega t$ and seek the solutions of (3) in the form

$$m_{\pm} = \{ m_{\pm}^{o} \exp(-i\Omega t) + m_{\mp}^{o*} \exp(i\Omega t) \} \exp(\kappa \mp igH_o)t ,$$

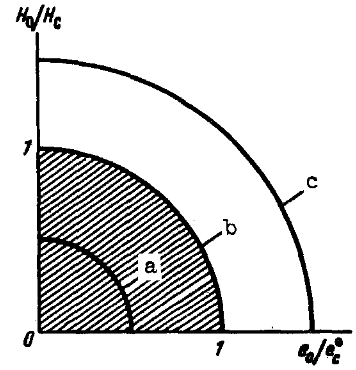
$$\ell_{\pm} = \{ \ell_{\pm}^{o} \exp(-i\Omega t) + \ell_{\mp}^{o*} \exp(i\Omega t) \} \exp(\kappa \mp igH_o)t .$$

Recognizing further that $gM_0\lambda\vec{e}_0 \ll \Omega$ (corresponding to fields $e_0 \ll 3 \times 10^6$ V/cm) and $\eta/\tau \ll \Omega$, we obtain from (3) the following expressions for Ω and κ:

$$\Omega^2 = (gM_o)^2 2\eta a, \qquad (4)$$

$$\kappa_{\pm} = -\frac{\eta}{\tau} \pm \left\{ \left(\frac{\eta}{\tau} \frac{H_o}{H_c} \right)^2 + \frac{1}{4} (g\lambda M_o e_o)^2 \right\}^{1/2}, \qquad (5)$$

where $H_c - \Omega g^{-1}$. We shall henceforth be interested in the low-frequency branch of the spin waves, characterized by the frequency $\omega = \Omega - gH_0$ and the increment κ_+, since this branch becomes unstable at smaller values of the electric field.



Growth increment of the spin wave as a function of the external fields H_0 and e_0. Curves a, b, and c correspond to increments $\kappa = -\gamma/2$, 0, and $\gamma/2$. The stability region is cross hatched.

According to (5) (see also the figure) spin waves will be excited at $\vec{e}_0 = \vec{e}_c^{\,0} \equiv 2\eta(gM_0\tau\lambda)^{-1}$ (in order of magnitude, $\vec{e}_c^{\,0} \sim 300$ V/cm) in the absence of an external magnetic field, if the ratio γ/Ω does not exceed 10^{-4} ($\gamma = \eta/\tau$ is the spin-wave damping decrement due to their interaction with one another, with phonons, and with inhomogeneities in the crystal). Near the overturning field, as $H_0 - H_c \sim 10^{-2}H_c$, the same electric field will lead to excitation of spin waves even in crystals with $\gamma/\Omega \sim 10^{-3}$.

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DETERMINATION OF THE MASS OF THE MUONIC NEUTRINO IN RADIATIVE PION DECAYS

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Until recently, the experimental limits on the muonic-neutrino mass m_ν were obtained from an analysis of the decays $\pi^+ \to \mu^+\nu_\mu$ and $\mu^+ \to e^+\nu_\mu\nu_e$. The