Being interested in parametric excitation of spin waves (see, e.g., [5]), we put $\vec{e}_p = \vec{e}_0 \cos 2\Omega t$ and seek the solutions of (3) in the form

$$m_\pm \{ m_\pm^0 \exp(-i\Omega t) + m_\mp^{0*} \exp(i\Omega t) \} \exp(\kappa \mp igH_0)t,$$

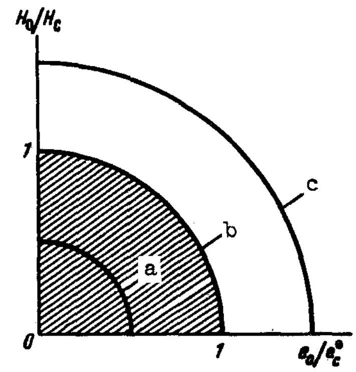
$$\ell_\pm = \{ \ell_\pm^0 \exp(-i\Omega t) + \ell_\mp^{0*} \exp(i\Omega t) \} \exp(\kappa \mp igH_0)t.$$

Recognizing further that $gM_0\lambda\vec{e}_0 \ll \Omega$ (corresponding to fields $e_0 \ll 3 \times 10^6$ V/cm) and $\eta/\tau \ll \Omega$, we obtain from (3) the following expressions for Ω and κ:

$$\Omega^2 = (gM_0)^2 2\eta a, \tag{4}$$

$$\kappa_\pm = -\frac{\eta}{\tau} \pm \left\{ \left(\frac{\eta}{\tau}\frac{H_0}{H_c}\right)^2 + \frac{1}{4}(g\lambda M_0 e_0)^2 \right\}^{1/2}, \tag{5}$$

where H_c - Ωg^{-1}. We shall henceforth be interested in the low-frequency branch of the spin waves, characterized by the frequency $\omega = \Omega - gH_0$ and the increment κ_+, since this branch becomes unstable at smaller values of the electric field.



Growth increment of the spin wave as a function of the external fields H_0 and e_0. Curves a, b, and c correspond to increments $\kappa = -\gamma/2$, 0, and $\gamma/2$. The stability region is cross hatched.

According to (5) (see also the figure) spin waves will be excited at $\vec{e}_0 = \vec{e}_c^0 \equiv 2\eta(gM_0\tau\lambda)^{-1}$ (in order of magnitude, $\vec{e}_c^0 \sim 300$ V/cm) in the absence of an external magnetic field, if the ratio γ/Ω does not exceed 10^{-4} ($\gamma = \eta/\tau$ is the spin-wave damping decrement due to their interaction with one another, with phonons, and with inhomogeneities in the crystal). Near the overturning field, as $H_0 - H_c \sim 10^{-2}H_c$, the same electric field will lead to excitation of spin waves even in crystals with $\gamma/\Omega \sim 10^{-3}$.

[1] G.A. Smolenskii and N.N. Krainik, Usp. Fiz. Nauk 97, 657 (1969) [Sov. Phys.- Usp 12, 271 (1969)].

[2] F.R. Morgenthaler, J. Appl. Phys. 36, 3102 (1965); Phys. Rev. Lett. 11, 69, 239(E) (1963).

[3] A.I. Akhiezer and I.A. Akhiezer, Zh. Eksp. Teor. Fiz. 59, 1009 (1970) [Sov. Phys.-JETP 32, 549 (1971)].

[4] A.I. Akhiezer, V.G. Bar'yakhtar, and S.V. Peletminskii, Spinovye volny (Spin Waves), Nauka, 1967.

[5] L.D. Landau and E.M. Lifshitz, Mekhanika (Mechanics), 2nd edition, Nauka, 1965, Sec. 27.

DETERMINATION OF THE MASS OF THE MUONIC NEUTRINO IN RADIATIVE PION DECAYS

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Until recently, the experimental limits on the muonic-neutrino mass m_ν were obtained from an analysis of the decays $\pi^+ \to \mu^+\nu_\mu$ and $\mu^+ \to e^+\nu_\mu\nu_e$. The best limitation, $m_\nu < 1.6$ MeV [1], was obtained by measuring the muon momentum in the decay $\pi^+ \to \mu^+\nu_\mu$. Further decrease of the limit on m_ν in the decay of a pion at rest requires not only an improvement of the accuracy with which the muon momentum was measured, but also a more accurate determination of the pion mass. The appreciable error in the determination of m_π, equal to 0.013 MeV[1]), does not make it possible to measure m_ν if it is smaller than 1 MeV. In the decay $\mu^+ \to e^+\nu_\mu\nu_e$, owing to the large energy release, events with low-energy ν_μ have low probability, and therefore the neutrino mass affects the energy spectrum of the positrons only in a very narrow region [3], and to improve the limit on m_ν it is necessary to measure this spectrum with presently-unattainable accuracy.

In view of the availability of meson factories, it will be possible in the nearest future to use the rare decays $\pi^+ \to \mu^+\nu_\mu\gamma$ and $\pi^+ \to \mu^+\nu_\mu e^+e^-$ in order to lower the limit of m_ν. The idea of using the $\pi^+ \to \mu^+\nu_\mu\gamma$ decay for this purpose was first advanced by Goldhaber [4]. He proposed to determine m_ν from the position of the end point of the γ-quantum spectrum, something that requires no theoretical assumptions whatever concerning the character of the interaction. It is desirable, however, to know also the form of the spectrum near the end point. It is known [5 - 6] that the main contribution to the total probability of this decay is made by internal bremsstrahlung (IB). However, structural decay (SD) may appear also in the differential probability. In [4], the contribution of IB to the energy spectrum of γ quanta has been estimated and it has been noted that its interference with SD can greatly alter the form of the spectrum near the end point, and by the same token influence the interpretation of the results. In [4] there was considered also the case when, besides registration of γ quanta (experiment 1), the muon momentum is measured with high accuracy (experiment 2).

In the present article we investigate in detail the spectra of γ-quantum energies K_0 in the decay $\pi^+ \to \mu^+\nu_\mu\gamma$ for experiments 1 and 2. We have taken the contribution of SD into account in the calculations, assuming for the parameter γ (the ratio of the axial form factor to the vector form factor, see [5]) the values obtained in experiment on the $\pi^+ \to e^+\nu_e\gamma$ decay [7]. It turns out that for any of the two values of γ the contribution of SD is smaller by two orders of magnitude than the contribution of IB, in the entire region of the K_0 spectrum, for both experiments in question. We assume that the V - A form of interaction is retained for a neutrino with $m_\nu \neq 0$. In this case the spectrum of the γ quantum is determined [8] by the product of the spectrum for $m_\nu = 0$ by the ratio of the phase volume with mass $m_\nu \neq 0$ to the phase volume with $m_\nu = 0$. In principle this can be verified by measuring the spectrum in a region far from the end point, where the influence of m_ν is negligibly small. Radiative corrections can also distort the form of the spectrum. One can expect, however, that in both experiments they can be neglected, since the kinetic energy of the muon produced in the decay of a π meson at rest is small ($\lesssim 4$ MeV).

For experiment 1 we calculated the energy spectrum of the γ quanta

$$R = \int_{E_{min}(k_o, m_\nu)}^{E_{max}(k_o, m_\nu)} \frac{10^8}{W_\pi} \frac{d^2 W_{IB}}{d(k_o/m_\pi)\, dE_\mu} \, dE_\mu, \tag{1}$$

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[1])The mass of the π^- meson is known from experiments on x rays in π-mesic atoms [2].

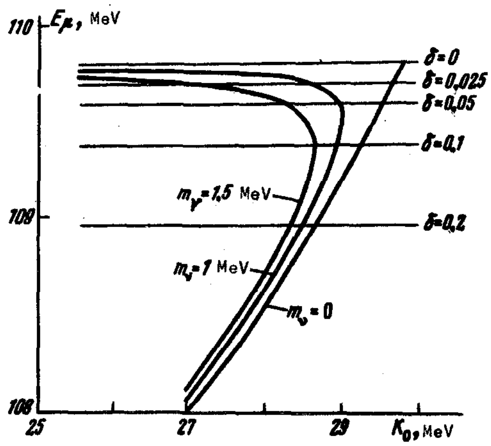

Fig. 1

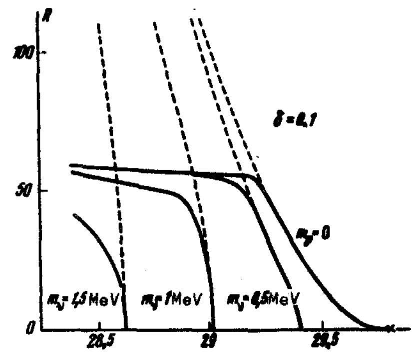

Fig. 2

where E_μ is the muon energy, W_π is the probability of the $\pi^+ \to \mu^+\nu_\mu$ decay, and $E_{min}(k_0, m_\nu)$ and $E_{max}(k_0, m_\nu)$ are the kinematic limits of integration with respect to the moun energy. Figure 1 shows plots of $E_{min}(k_0, m_\nu)$ and $E_{max}(k_0, m_\nu)$ for different values of m_ν. If besides registering the γ quanta we measure the muon energy and retain events with kinetic energy $T > \bar{T} = T^0_{max}(1 - \delta)$, where T^0_{max} is the maximum kinetic energy of the muon at $m_\nu = 0$ and δ is the accuracy of its measurement, then we should use as the upper limit in (1) the expression

$$E_{min} = max\,\{\bar{E},\ E_{min}(k_0,\ m_\nu)\}, \tag{2}$$

where $\bar{E} = \bar{T} + m_\mu$.

The horizontal lines in Fig. 1 represent $\bar{E}$ for different values of δ (δ = 0.025, 0.05, and 0.1). At the maximum photon energy

$$k_0^{max} = \frac{1}{2\,m_\pi}\,[m_\pi^2 - (m_\mu + m_\nu)^2] \tag{3}$$

the muon energy is

$$E_M = m_\mu\,\frac{m_\pi^2 + (m_\mu + m_\nu)^2}{2m_\pi(m_\mu + m_\nu)}\ , \tag{4}$$

and the muon and the neutrino have the same velocity v. It is seen from Fig. 1 that if $\bar{E} < E$, then there exists a region near k_0^{max} where $E_{min} = E_{min}(k_0, m_\nu)$, and thus the spectra for experiments 1 and 2 coincide. With increasing distance from k_0^{max}, E_{min} becomes equal to $\bar{E}$, and the spectrum of the γ quanta for the experiment 2 becomes different from the spectrum for experiment 1. Figure 2 shows the spectra for experiment 2 at δ = 0.1, and m_ν = 1.5, 1, 0.5, and 0 MeV. The dashed lines denote the spectra for experiment 1. It is seen from Fig. 2, that in the case of experiment 2, the spectra acquire a plateau that terminates sharply near k_0^{max}. It is clear that when the spectra have such a form the influence of the inaccuracy in the determination of the energy k_0 is

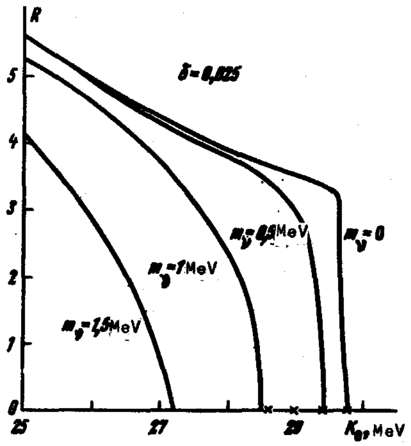

Fig. 3

less significant. However, the integral probability decreases somewhat in the region of interest to us.

If the muon registration threshold is $\bar{E} > E_{\parallel}$, then the end point in the γ-quanta spectrum shifts by an amount

$$\Delta(\bar{E}, m_\nu) = k_0^{max} - \frac{1}{2}\,\frac{m_\pi^2 + m_\mu^2 - m_\nu^2 - 2m_\pi \bar{E}}{m_\pi - \bar{E} - \sqrt{\bar{E}^2 - m_\mu^2}}\,. \tag{5}$$

This shift is the larger, the larger m_ν and the larger E. In principle, this makes it possible to use instruments with worse resolution with respect to k_0. However, the level at which it is necessary to measure the probability of the process in this case is exceedingly small. Figure 3 shows the spectrum of the γ quanta at $\delta = 0.025$ and $m_\nu = 1.5$, 1, 0.5, and 0 MeV. The symbol x marks the undisplaced positions of the end points.

In [4] it is proposed to measure the momentum of the muon in experiment 2 with accuracy 0.1 MeV/c, corresponding to an accuracy ∿0.7% in the measurement of the kinetic energy. For this case, the estimated integral probability in an interval of 3 MeV away from the end point of the k_0 spectrum is $\sim 5 \times 10^{-8}$ W_π.

Our calculations show that the integral probability interval is $\sim 10^{-9}$ W_π already at $\delta = 0.025$.

We have considered also the process $\pi^+ \to \mu^+\nu_\mu e^+e^-$, which in principle can be used to determine the mass of muonic neutrinos. This decay was discussed for the same purpose in [9], where the contribution of IB to the spectrum of the summary energy of the e^+e^- pair was calculated.

We have chosen the matrix element of the process in the same form as in [10]. It turns out that the contribution of SD is negligibly small. A comparison of the integral probability at the tails of the k_0 spectra in the decays $\pi^+ \to \mu^+\nu_\mu\gamma$ and $\pi^+ \to \mu^+\nu_\mu e^+e^-$ has shown that the process $\pi^+ \to \mu^+\nu_\mu\gamma$ is more convenient for the determination of the mass m_ν, even if thin converters with registration efficiency $\sim 10^{-2}$ are used for precision measurement of the γ-quantum energy. For this reason, we do not present the spectra for the decay $\pi^+ \to \mu^+\nu_\mu e^+e^-$, although our results differ from the corresponding results of [9]. The total probability of the $\pi^+ \to \mu^+\nu_\mu e^+e^-$ decay calculated by us is given in [10].

In conclusion, we are grateful to S.M. Bilen'kii and S.M. Korenchenko for a discussion of the results.

[1] P.S.L. Booth, R.G. Johnson, E.G.H. Williams, and J.R. Wormald, Phys.Lett. 26B, 39 (1967).
[2] R.E. Shafer, Phys. Rev. 163, 1451 (1967).
[3] G.R. Allcoch, Proc. Phys. Soc. 85, 875 (1965).
[4] A.S. Goldhaber, Phys. Rev. 130, 760 (1963).
[5] S.G. Brown and S.A. Bludman, Phys. Rev. 136, B1160 (1964).

[6] M.G. Smoes, Nucl. Phys. B20, 237 (1970).
[7] P. Depommier, J. Heintze, C. Ruboia, and V. Soergel, Phys. Lett. 7, 285 (1963).
[8] P. Dennery and H. Primakoff, Phys. Lett. 6, 67 (1963).
[9] S.A. Pikin and Yu.N. Kharkats, Yad. Fiz. 1, 291 (1965) [Sov. J. Nuc. Phys. 1, 207 (1965)].
[10] D.Yu. Bardin, S.M. Bilen'kii, G.V. Mitsel'makher, and N.M. Shumeiko, JINR Preprint R2-5520, Dubna, 1970.

DUAL AMPLITUDE FOR PROCESSES WITH PARTICIPATION OF PHOTONS

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There is no doubt that a very important problem in elementary particle physics is the construction of the dual amplitude containing besides the hadrons photons and (or) leptons. In this paper we attempt to obtain a dual amplitude for the process N mesons + γ. The analysis was carried out in terms of the approach [1, 2] based on the parton model. The obtained amplitude is compared with the Veneziano amplitude [3], which was used earlier [4] to describe the processes $\gamma\pi \to 2\pi$. One of the interesting properties of this approach, in our opinion, is the possibility of carrying out an analogy with the vector dominance model (VDM).

We assume that the partons are vector charged particles. We choose the following interaction Lagrangians: $\lambda\varepsilon_{\alpha\beta\gamma\delta}\phi_\alpha\phi_\beta\phi_\gamma\phi_\delta$ for the parton-parton field, $e\varepsilon_{\alpha\beta\gamma\delta}A_\alpha(\phi_\beta\partial_\gamma\phi^*_\delta - \partial_\gamma\phi_\beta\phi^*_\delta) + eA_\alpha\phi_\alpha$ for the parton and electromagnetic field, and $\lambda\partial_\mu M\phi_\nu\phi_\alpha\phi_\beta\varepsilon_{\mu\nu\alpha\beta}$ for the parton field and the field of the spinless mesons; here ϕ_α, A_α, and M are respectively the parton, electromagnetic, and meson fields. If, following [2], we write the amplitude corresponding to Fig. 1 and then expand $\ln\{P[(X^2)]\}$ ($P[(X^2)]$ is the parton propagator) near $X^2 \sim 0$, then going to the limit as $n \to \infty$, where n is the number of vertices of the diagram of Fig. 1, we obtain

$$M = f_{ij}\epsilon_i p_j A,$$
$$A = C\delta\Big(\sum_{j=1}^{N} k_j + p\Big)\exp\{-L(z_1, \ldots, z_N, z_\nu)\}, \quad (1)$$

where $L(z_1, \ldots, z_N, z_\nu)$ is the Lagrangian of a liquid flowing through a flat disk of unit radius with apertures at the points z_i and z_ν ($z_i = \exp[i\theta_i]$ $z_\nu = \rho\exp[i\theta]$, $\theta_i \geq \theta_{i+1}$), the liquid current being k_i in aperture z_i and p in z_ν; $f_{ij} = -f_{ji}$. The latter condition guarantees gauge invariance of the amplitude. The Lagrangian L will be represented in the form

$$L(z_1, \ldots, z_N, z_\nu) = \sum_{i<j} R_{ij}k_i k_j + \sum_{j=1}^{N} R_{j\nu}k_j p, \quad (2)$$

where $R_{\alpha\beta}$ is the resistance of the disk between the points z_α and z_β, equal to

$$R_{\alpha\beta} \sim \ln|z_\alpha - z_\beta| \quad \text{при } z_\alpha \to z_\beta. \quad (3)$$