HIGH-ENERGY ASYMPTOTIC CROSS SECTIONS FOR THE PRODUCTION OF e^+e^- PAIRS IN COLLISIONS OF HEAVY CHARGED PARTICLES

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The cross section for the production of an e^+e^- pair in the collision of two fast charged particles, in the limit of high energy E in the c.m.s. of the colliding particles, was obtained in 1934 by Landau and Lifshitz [1], who used the method of equivalent photons. They have shown that the cross section is proportional to $\ln^3 E$. In view of the fact that it will become possible in the nearest future to verify experimentally the cross sections for electroproduction of pairs in collisions of high energy e or p beams with a target, the question arises of the limits of applicability of the formula derived by Landau and also of its refinement.

In this paper we obtain the cross section for the production of an e^+e^- pair in collisions between two heavy charged particles in the limit of high c.m.s. energies, accurate to terms $\sim\ln^2 E$. It turns out that the term $\sim\ln^2 E$ enters in the cross section with a large negative coefficient, so that at energies $E \lesssim (6 - 7)$ GeV the cross section becomes negative if only these two terms are taken into account. We hope to generalize in the nearest future the expressions for the cross section of pair production with accuracy to terms $\sim 1/E$.

Asymptotically the main contribution to the cross section comes from diagrams of the type of Fig. a. (A shaded block in Fig. 1 corresponds to scattering of light by light in the lowest order of perturbation theory.) Indeed, diagram b contains fermions in the t channel, and therefore its contribution is smaller by a factor $s = (p_1 + p_2)^2$ than that of diagram a [2]. The amplitude of diagram c, although not small when $s \to \infty$, it contains an extra $1/M^2$ compared with diagram a. (We assume that $m_e = 1$).

We expand the momenta k_1 and k_2, following Sudakov [3]

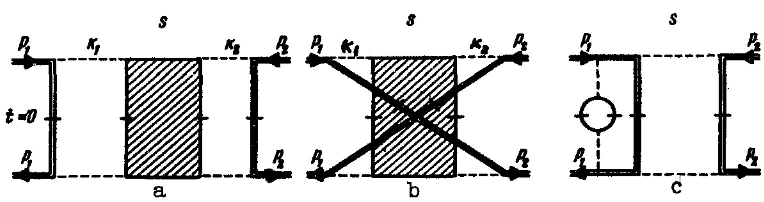
$$k_{1,2} = \alpha_{1,2} p_2' + \beta_{1,2} p_1' + k_{1,2\perp}, \quad p'^2 = p_1'^2 = O(M^4/s^2),$$

$$p_{1,2} k_\perp = p'_{1,2} k_\perp = 0.$$

Then the imaginary part in the s channel of the amplitude of diagram a at t = 0 is written in the form

$$\int \frac{\frac{s}{2} d\alpha_1 d\beta_1 d^2 k_{1\perp} \delta_1 \theta_1}{(k_{1\perp}^2 + s\alpha_1\beta_1)^2} \int \frac{\frac{s}{2} d\alpha_2 d\beta_2 d^2 k_{2\perp} \delta_2 \theta_2}{(k_{2\perp}^2 + s\alpha_2\beta_2)^2} N_{\mu\nu\rho\sigma} f^{\mu\nu\rho\sigma}, \qquad (1)$$

where $\delta_{1,2}$ $\theta_{1,2}$ - δ and θ- are functions corresponding to the division of the lines of the diagram of Fig. a, corresponding to the nucleons;



$$N_{\mu\nu\rho\sigma} = \frac{1}{4} \sum_{\lambda\lambda'} \bar{u}^{\lambda}(p_1)\,\gamma_\mu(p_1 - k_1 + m)\,\gamma_\nu\, u^{\lambda}(p_1)\,\bar{u}^{\lambda'}(p_2)\,\gamma_\rho(p_2 - k_2 + m)$$

$$\times \gamma_\sigma u^{\lambda'}(p_2) = 4\,\mathrm{Sp}\,\gamma_\mu(p_1 - k_1 + m)\gamma_\nu(p_1 - m)\,\mathrm{Sp}\,\gamma_\rho(p_2 - k_2 + m)$$

$$\times \quad \gamma_\sigma(p_2 - m) \sim p_{1\mu} p_{1\nu} p_{2\rho} p_{2\sigma} \qquad (2)$$

$f^{\mu\nu\rho\sigma}$ is the amplitude for scattering of light by light off the mass shell and satisfies the gauge-invariance condition

$$k_{1\mu} f^{\mu\nu\rho\sigma} = k_{1\nu} f^{\mu\nu\rho\sigma} = k_{2\rho} f^{\mu\nu\rho\sigma} = k_{2\sigma} f^{\mu\nu\rho\sigma} = 0. \qquad (3)$$

From the limitations imposed on δ_1 and θ_1 (δ_2 and θ_2) it follows that the momentum k_1 (k_2) has a large component with respect to p_1 (p_2): $0 < \beta_1(\alpha_2) < 1$; $\alpha_1(\beta_2) = O(M^2/s)$. Using this, and also (2) and (3), we can write for the product $N_{\mu\nu\rho\sigma} f^{\mu\nu\rho\sigma}$

$$N_{\mu\nu\rho\sigma} f^{\mu\nu\rho\sigma} \sim p_{1\mu} p_{1\nu} p_{2\rho} p_{2\sigma} f^{\mu\nu\rho\sigma} = s^2 \frac{f_{\perp}^{\mu\nu\rho\sigma} k_{1\perp\mu} k_{1\perp\nu} k_{2\perp\rho} k_{2\perp\sigma}}{(s\,\alpha_2\beta_1)^2}, \qquad (4)$$

where $f^{\perp}$ denotes the part of the tensor f transverse to p_1 and p_2.

We integrate in (1) with respect to α_1 and β_2 with the aid of δ-functions, and write the integration with respect to α_2 and β_1 in the form

$$s \int d\alpha_2 \int d\beta_1 = \int ds_1 \int_{s_1/s}^{1} d\beta_1/\beta_1 \,, \qquad s_1 = s\alpha_2\beta_1 \,. \qquad (4a)$$

Further, we break up the region of integration with respect to $k_{\perp 2,1}$ into two regions, $0 < |k^2_{\perp 1,2}| < \sigma$ and $\sigma < |k^2_{\perp 1,2}| < \infty$ (σ is a numerically small quantity not connected with s). Using the symmetry of the integrand in (1) with respect to the substitution $p_1 \leftrightarrow p_2$, we represent the integrations with respect to $k_{1\perp}$ and $k_{2\perp}$ ($k^2_{1,2\perp} = -k^2_{1,2}$ and $k_{1,2}$ is assumed to be Euclidean) in the form

$$\int_0^\infty dk_1^2 \int_0^\infty dk_2^2 = -\int_0^\sigma dk_1^2 \int_0^\sigma dk_2^2 + 2\int_0^\sigma dk_1^2 \int_0^\infty dk_2^2 + \int_\sigma^\infty dk_1^2 \int_\sigma^\infty dk_2^2 = -I_1 + 2I_2 + I_3 \,. \qquad (5)$$

Then obviously, in calculating I_1 in (5) accurate to terms $\sim\delta$, we can put in lieu of $f_\perp(s_1, k_1^2, k_2^2)$ the function $f_\perp(s_1, 0, 0)$, which in accordance with the optical theorem is connected with the cross section for the production of an e^+e^- pair by two photons [3]: $(1/4)\,\mathrm{Im}\, f_\perp^{\mu\mu\nu\nu}(s, 0, 0) = (s/4\pi)\sigma_p(s)$. Using this, and also (4) and (4a), we rewrite I_1 in the form

$$I_1 \sim s \int_4^\infty ds_1 \frac{\sigma_p(s_1)}{s_1} \int_{s_1/s}^{1} \frac{d\beta_1}{\beta_1} \int_0^\sigma \frac{dk_1^2\, k_1^2}{(k_1^2 + \beta_1^2)^2} \int_0^\sigma \frac{dk_2^2\, k_2^2}{(k_2^2 + s_1^2/s^2\beta_1^2)^2}$$

$$= s\left[A_1 \ln^3 s + B_1 \ln^2 s + \ln\frac{\sigma}{M^2}(s)\right] \qquad (6)$$

where

$$a = \int_4^{\infty} \frac{ds_1}{s_1}\,\sigma_p(s_1) = \frac{14}{9}\,\pi r_o^2, \quad b = \int_4^{\infty} \frac{ds_1}{s_1}\,\ln s_1 \sigma_p(s_1) = \frac{43}{9}\,\pi r_o^2,$$

$$A_1 = \frac{2}{3}\,a, \quad B_1 = -\,2a - 2b\,. \tag{6a}$$

The terms in (6) containing $\ln(\sigma/M^2)$, cancel out upon addition with analogous terms that appear in the calculation of I_2 and I_3 in (5). We shall not present here a proof of this statement, since it will greatly increase the volume of the article.

Analogously, using the Racah formula in the calculation of I_2 [4], for the cross section for the production of a pair by a photon and a nucleus [to find the coefficient of $\ln^2 s$ in this problem it suffices to use for large s its asymptotic expression [4, 3]

$$\sigma_p^{\gamma Y}(s) \sim r_o^2\,\alpha\left(\frac{28}{9}\ln s - \frac{218}{27}\right)]\,,$$

we obtain

$$I_2 \sim s\int_{s_o}^{s} \frac{ds_1}{s_1}\left(\frac{28}{9}\ln s_1 - \frac{218}{27}\right)\left(\ln\frac{s^2\sigma}{M^2 s_1^2} - 1\right)$$

$$= \left[A_2 \ln^3 s + B_2 \ln^2 s + O\left(\ln\frac{\sigma}{M^2}\right)\right]s \tag{7}$$

$$A_2 = \frac{28}{27}, \quad B_2 = -\,\frac{260}{27}\,. \tag{7a}$$

The contribution of the integral I_3 contains only terms $\sim s(A_3 \ln s + \text{const})$. Substituting (5), (6), and (7) in (1) and using the optical theorem, we obtain for the pair-production cross section

$$\sigma = \frac{r_o^2 Z_1^2 Z_2^2 \alpha^2}{27\pi}\,[\,28\ln^3 s - 178\ln^2 s\,]. \tag{8}$$

The calculation of the contribution of the terms $\sim \ln s$ and const to the cross section called for a more detailed analysis, which apparently can be carried through to conclusion only with the aid of a computer.

In conclusion, we take the opportunity to thank G.V. Frolov, L.N. Lipatov, and V.G. Gorshkov for suggesting the problem, for constant stimulating interest during the course of performance of the work, and for numerous critical remarks.

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SELF-FOCUSING OF POWERFUL SOUND DURING THE PRODUCTION OF BUBBLES

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The production of bubbles in a liquid under the influence of powerful acoustic and ultrasonic waves can greatly increase the nonlinear processes of refraction, such as self-focusing, self-defocusing, beam rotation, etc. In the most interesting liquids (water) the appearance of bubbles is connected with release of dissolved gas as a result of the decreased pressure in the sound wave, jolting, or slight heating - factors that decrease the solubility of the gas in the water. The appearance of bubbles greatly increases the compressibility K of the water and decreases the speed of sound $c_s = 1/\sqrt{K\rho}$, if the sound frequency ω is lower than the resonant frequency of the bubbles $\omega_r = (1/a)\sqrt{3\gamma p/\rho}$, where a is the bubble radius, p the gas pressure, and ρ the density of the liquid. Indeed, in the adiabatic case for a gas we have $\partial v/\partial p \simeq v/\gamma p$; from this we obtain immediately the compressibility of water in the presence of bubbles $K' = K + (4\pi/3\gamma)(Na^3/p)$, where N(a) is the concentration[1] of the bubbles of radius a, and the density is $\rho' = \rho[1 - (4\pi/3)Na^2]$. Since $pK \simeq 10^{-4} \ll 1$, the change of density can be neglected, and therefore

$$c_s^2 = c_{s0}^2/[1 + (4\pi/3\gamma)(Na^3/pK)].$$

In the case of a small change of the velocity $c_s \simeq c_{s0}[1 - (2\pi Na^3/3\gamma pK)]$ or $\Delta c_s/c_s \simeq -2\pi Na^3/3\gamma pK$; we see that in this case the conditions of self-focusing of the sound are satisfied, since the concentration of the bubbles is usually larger where the amplitudes of the sound wave $N = N(A_s)$ is larger, and the dependence can be quite strong when the amplitude of the pressure is of the order of one atmosphere (e.g., for ordinary water).

The self-focusing conditions are determined from the ratio of the compensation of the divergence angle $\theta \simeq \sqrt{\delta c_s/c_s} \simeq [Na^3/pK]^{1/2}$; (the threshold corresponds to the diffraction divergence $\theta_D \sim \lambda_s/D$, where D is the beam diameter). With this, one can ensure small scattering of sound by the bubbles $\Sigma_s L \simeq N4\pi a^2(\omega/\omega_r)^4 L \ll 1$ along the path L where the self-focusing is realized ($L \simeq D/\sqrt{\delta c_s/c_s}$ at beyond-threshold conditions.

We note that if the sound frequency becomes commensurate with the resonant frequency of the bubble, then the changed velocity is

$$1/c_s^2 = 1/c_{s0}^2 + 4\pi a N/\omega^2[(\omega_r/\omega)^2 - 1 - ika]$$

i.e., when $\omega > \omega_r$, the speed of sound increases in the presence of bubbles and defocusing should take place. As $\omega \to \omega_r$, the nonlinearity increases sharply, but the scattering of the sound also increases ($\sigma_{sr} \sim \lambda_s^2/\pi$). Since the effects considered above depend on the bubble-production processes, they can depend on the purification and degassing of the liquid, on the gas content, on the pressure, on the supersaturation, on the closeness of the liquid to the boiling

[1]For simplicity we assume that the distance between the bubbles is much smaller than the sound wavelength λ_s.