SELF-FOCUSING OF POWERFUL SOUND DURING THE PRODUCTION OF BUBBLES

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The production of bubbles in a liquid under the influence of powerful acoustic and ultrasonic waves can greatly increase the nonlinear processes of refraction, such as self-focusing, self-defocusing, beam rotation, etc. In the most interesting liquids (water) the appearance of bubbles is connected with release of dissolved gas as a result of the decreased pressure in the sound wave, jolting, or slight heating - factors that decrease the solubility of the gas in the water. The appearance of bubbles greatly increases the compressibility K of the water and decreases the speed of sound $c_s = 1/\sqrt{K\rho}$, if the sound frequency ω is lower than the resonant frequency of the bubbles $\omega_r = (1/a)\sqrt{3\gamma p/\rho}$, where a is the bubble radius, p the gas pressure, and ρ the density of the liquid. Indeed, in the adiabatic case for a gas we have $\partial v/\partial p \approx v/\gamma p$; from this we obtain immediately the compressibility of water in the presence of bubbles $K' = K + (4\pi/3\gamma)(Na^3/p)$, where N(a) is the concentration[1] of the bubbles of radius a, and the density is $\rho' = \rho[1 - (4\pi/3)Na^2]$. Since $\rho K \approx 10^{-4} \ll 1$, the change of density can be neglected, and therefore

$$c_s^2 = c_{s0}^2/[1 + (4\pi/3\gamma)(Na^3/pK)].$$

In the case of a small change of the velocity $c_s \approx c_{s0}[1 - (2\pi Na^3/3\gamma pK)]$ or $\Delta c_s/c_s \approx -2\pi Na^3/3\gamma pK$; we see that in this case the conditions of self-focusing of the sound are satisfied, since the concentration of the bubbles is usually larger where the amplitudes of the sound wave $N = N(A_s)$ is larger, and the dependence can be quite strong when the amplitude of the pressure is of the order of one atmosphere (e.g., for ordinary water).

The self-focusing conditions are determined from the ratio of the compensation of the divergence angle $\theta \approx \sqrt{\delta c_s/c_s} \approx [Na^3/pK]^{1/2}$; (the threshold corresponds to the diffraction divergence $\theta_D \sim \lambda_s/D$, where D is the beam diameter). With this, one can ensure small scattering of sound by the bubbles $\Sigma_s L \approx N4\pi a^2(\omega/\omega_r)^4 L \ll 1$ along the path L where the self-focusing is realized ($L \approx D/\sqrt{\delta c_s/c_s}$ at beyond-threshold conditions.

We note that if the sound frequency becomes commensurate with the resonant frequency of the bubble, then the changed velocity is

$$1/c_s^2 = 1/c_{s0}^2 + 4\pi a N/\omega^2[(\omega_r/\omega)^2 - 1 - ika]$$

i.e., when $\omega > \omega_r$, the speed of sound increases in the presence of bubbles and defocusing should take place. As $\omega \to \omega_r$, the nonlinearity increases sharply, but the scattering of the sound also increases ($\sigma_{sr} \sim \lambda_s^2/\pi$). Since the effects considered above depend on the bubble-production processes, they can depend on the purification and degassing of the liquid, on the gas content, on the pressure, on the supersaturation, on the closeness of the liquid to the boiling

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[1] For simplicity we assume that the distance between the bubbles is much smaller than the sound wavelength λ_s.

point, and on other parameters that can be used to enhance or weaken the processes, or to control them. For prolonged processes, the bubbles may float out of the beam or be removed from it by the beam pressure gradient.

Different combined nonlinear effects of action of one sound beam on another or the action of light on sound and of sound on light via bubble formation are also possible. (These effects were studied by us jointly with T.G. Rakhmanina and will be reported later.)

The aforementioned nonlinear properties connected with cavitation may exceed by many times the heating nonlinearity [1] connected with sound absorption and ensure self-focusing of ultrasound and sound waves.

We note that at large amplitudes, nonlinearity of the bubble oscillations may appear and the dependence of the velocity on the sound amplitude may be determined not only by the dependence of the bubble concentration on the sound amplitude.

The bubble instability considered above differs noticeably from the nonlinearity due to heating during absorption, which appears only for hypersound and ultrasound of very high frequency.

For ordinary ultrasound and sound, the absorption is small, and the diffraction divergence large, so that a large nonlinearity is needed in order to overcome the divergence. It is precisely the bubble instability which can ensure self-focusing of such radiation, in view of the large values of the nonlinearity.

[1] G.A. Askar'yan, ZhETF Pis. Red. 4, 144 (1966) [JETP Lett. 4, 99 (1966)].

ERRATUM

The article by D.V. Gol'tsov and V.V. Skobelev, V. 13, No. 3, p. 122 contains an error. In calculating the matrix element in the Furry picture, no account was taken of the contribution due to the phase factor exp i (x, 7), which enters in the expression for the Green's function of an electron in a homogeneous field, G(x, y) = exp i (x, y)S(x - y). This contribution may turn out to be equal in magnitude and opposite in sign to the function S(x - y). Thus, in the approximation considered, the matrix element of the process vanishes. A similar error was made also in the papers cited in our article [5, 6]. A corrected version of the article has been submitted to Vestnik Moskovskogo Universiteta (Herald of the Moscow University).