

The excess current in superconducting point junctions

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(Submitted 13 October 1978)

Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 10, 637-640 (20 November 1978)

It is shown theoretically that at high voltages $V \gg \Delta/e$ the current in the junctions is the sum of the ohmic current V/R and of the excess current I_{exc} . The obtained value $I_{\text{exc}} = (\pi^2/4 - 1)\Delta/2eR$ agrees with the experimental data.

PACS numbers: 74.50. + r

The theory of the Josephson effects has been developed to a sufficient extent only for tunnel junctions. Effects observed in other types of weakly-linked superconductors have much in common with those observed and investigated in tunnel junctions. There are also, however, substantial differences. It appears that the most pronounced difference is that the current-voltage characteristic $I(V)$ of the junctions and bridges exhibit an excess current (see, e.g., Refs. 1-5 and the literature cited therein), i.e., at large V the function $I(V)$ takes the form $I(V) = V/R + I_{\text{exc}} \operatorname{sgn} V$, where the excess current I_{exc} is a constant independent of V (in a wide range of V). Although the current I_{exc} was observed long ago, it has received in fact no theoretical explanation. If we disregard purely phenomenological models, the only ones who proposed a mechanism for the excess current were Likharev and Yakobson⁶, who considered a junction of length $d \ll \xi(T)$, [here $\xi(T)$ is the coherence length]. However, the mechanism proposed for I_{exc} in Ref. 6 is essentially connected with the use of the nonstationary Ginzburg-Landau equations, which are valid only for zero-gap superconductors. In addition, the excess current in Ref. 6 appears only when the current through the junction greatly exceeds the critical current I_c , namely at $I \approx I_c/\lambda$, where $\lambda = [d/\xi(T)]^2 \ll 1$ is the parameter that characterizes the weakness of the coupling. In the limit as $\lambda \rightarrow 0$ there is no excess current in this model. In the present paper, on the basis of microscopic equations, we calculate the current in a point junction (or in a bridge of variable thickness) of length $2d \ll \xi(T)(1 - T/T_c)^{1/4}$ and show that the $I(V)$ function contains an excess current.

Consider a short junction of an ordinary superconductor with a gap. We assume that the superconductor is dirty ($\tau T \ll 1$) and use the one-dimensional model considered in Refs. 6 and 7. In this model it is assumed that the junction is a thin superconducting filament of length $2d$, connecting two bulky superconductors (banks). In the banks ($x = \pm d$) all the functions are assumed to be in equilibrium and to correspond to phases $\pm \phi(t)/2$ and to potentials $\pm [v = (e/2)V = \partial \phi / \partial t]$. We use the equations for the matrix Green's functions \hat{g} and $\hat{g}^{R(A)}$ integrated with respect to $(p - p_F)p/m$.⁸⁻¹⁰ In our case of a short junction, the largest terms in these equations are those containing the spatial derivative

$$(p/m) \vec{\nabla} \hat{g} = 0, \quad (p/m) \vec{\nabla} \hat{g}^{R(A)} = 0, \quad (1)$$

where $\hat{g} = (\hat{g}_x, 0, 0)$ is the vector part of \hat{g} and determines the current in the junction [$\hat{g} = \hat{g}_0 = (p/p)\hat{g}$]. To determine the functions \hat{g} and $\hat{g}^{R(A)}$, which do not depend on x , we use the orthogonality conditions¹⁰

$$\hat{g}^{(R)} \hat{g}^{(R)} = \hat{1}; \quad \hat{g}^{(R)} \hat{g} + \hat{g} \hat{g}^{(A)} = 0 \quad (2)$$

and the boundary conditions $\hat{g}_{x=\pm d}^R = \hat{S}(t) \hat{g}_{eq}^R \hat{S}^+(t')$, where \hat{g}_{eq}^R is the equilibrium retarded Green's function, and the matrix $[S(t)]_{x=\pm d} = \frac{1}{2}[\hat{1} \cos(vt) \pm i\hat{\sigma}_2 \sin(vt)]$ takes into account the presence of the potential. The product of the functions in (2) is to be taken to mean a convolution with respect to the time variable. The boundary condition for the function \hat{g} is that it coincides with the regular part $\hat{g}^{(r)}$ at $x = \pm d$, since the anomalous part $\hat{g}^{(a)}$ differs from zero only in the case of deviation from equilibrium. For the Fourier components of $\hat{g}_x^{R(A)}$ it is easy to obtain from (1)

$$\hat{g}_x^R = -\frac{l}{2d} \operatorname{arcsinh} (2 \hat{g}_+^R \hat{g}_-^R) = -\frac{l}{2d} \sum_{k=0}^{\infty} a_k (2 \hat{g}_+^R \hat{g}_-^R)^{2k+1}, \quad (3)$$

where $\hat{g}_{\pm}^R = \frac{1}{2}[\hat{g}_{x=d}^R \pm \hat{g}_{x=-d}^R]$, and l is the mean free path. We seek \hat{g} in the form $\hat{g} = \hat{g}^R \hat{F} - \hat{F} \hat{g}^{(A)}$. Writing down the second condition of (2) at the points $x = \pm d$, we obtain after simple transformations

$$(\hat{g}_+^R \hat{F} - \hat{F} \hat{g}_-^{(A)} - \hat{g}^{(r)})_{x=\pm d} = 0. \quad (4)$$

As seen from the expansion of \hat{g}_x^R in powers of $(\hat{g}_+^R, \hat{g}_-^R)$, to determine the current

$$I = -\frac{ep}{12\pi} \operatorname{Sp} \int \frac{d\epsilon}{2\pi} (\hat{\sigma}_z \hat{g}_x) \quad (5)$$

we must calculate terms of the type

$$(\hat{g}_+^R \hat{g}_-^R)^{2k+1} \hat{F} - \hat{F} (\hat{g}_+^A \hat{g}_-^A)^{2k+1}. \quad (6)$$

Terms of this kind can be calculated with the aid of (3) and (4). Proceeding in this manner, we obtain a series for \hat{g}_x . We write down its "anomalous" part $\hat{g}^{(a)}$, which

determines the sought quasiparticle current

$$\hat{g}_x^{(a)} = -\frac{i}{2d} \sum_{k=0}^{\infty} a_k \sum_{n=0}^{2k} (\hat{g}_+^R \hat{g}_-^R)^{2k-n} \hat{A} (\hat{g}_+^A \hat{g}_-^A)^n, \quad (7)$$

where

$$\hat{A} = [(\hat{g}_+^R)^2 \hat{\sigma}_z - \hat{g}_+^R \hat{\sigma}_z \hat{g}_+^A + \hat{g}_-^R \hat{\sigma}_z \hat{g}_-^A - \hat{\sigma}_z (\hat{g}_-^A)^2] [\text{th } \beta(\epsilon + v) - \text{th } \beta(\epsilon - v)],$$

$\beta = 1/2T$. The sums in (7) can be calculated assuming large voltages $v > \Delta$. Then, summing the terms that are principal with respect to the parameter Δ/v , we obtain

$$I_{qp} = \frac{1}{4R} \int d\epsilon N_s(\epsilon + v) N_s(\epsilon - v) 2[D(\epsilon + v) - D(\epsilon - v) - 1] \times \left\{ \text{th } \beta(\epsilon + v) - \text{th } \beta(\epsilon - v) \right\}, \quad (8)$$

where $D(\epsilon) = (f^R - f^A)^{-1} [\text{Arsinh } f^R - \text{Arsinh } f^A]$, $f^{R(A)}$ is the equilibrium retarded (advanced) Gor'kov Green's function with allowance for the damping, $N_s = g^R - g^A$, and $g^R = (\epsilon/\Delta) f^R$. The expression for the current 8 differs from the corresponding expression in the case of a tunnel junction by the factor in the square brackets, which has a singularity at $|\epsilon \pm v| < \Delta$ (allowance for the damping eliminates this singularity). Therefore even at $T = 0$ the integrand differs from zero at all energies. The reason is that at the narrowest spot of the junction, where the gap oscillates with frequency $2eV > \Delta$, there is no time for the quasiparticle spectrum to be established, so that the contribution to I_{qp} is made by states corresponding to energies below the gap. Integrating, we get

$$I_{qp} R = V + \nabla \left(\frac{\pi^2}{4} - 1 \right) \text{th } (eV/2T).$$

This expression is valid at $eV > \Delta$ and at all temperatures. The excess current, which is determined by the second term, is of the order of the critical current I_c at low temperatures [$I_c R \approx \Delta$ at $T \ll \Delta$ (Ref. 7)] and exceeds I_c near T_c [$I_c R = (\pi \Delta^2/4T)$ at $T \gg \Delta$ (Ref. 7)]. The obtained temperature dependence and magnitude of the current I_{exc} agree with the experimental values obtained with point junctions.⁵

¹J.I. Pankove, Phys. Lett. **21**, 406 (1966).

²A.I. Akimenko, V.S. Solov'ev, and I.K. Yanson, Fiz. Nizk. Temp. **2**, 480 (1976) [Sov. J. Low Temp. Phys. **2**, 238 (1976)].

³M. Octavio, W.J. Skocpol, and M. Tinkham, Phys. Rev. B **17**, 159 (1978).

⁴V.N. Gubankov, V.P. Koshelets, and G.A. Ovsyannikov, Zh. Eksp. Teor. Fiz. **73**, 1435 (1977) [Sov. Phys. JETP **46**, 755 (1977)].

⁵Yu. Ya. Divin and F. Ya. Nad', Fiz. Nizk. Temp. **4**, 1105 (1978) [Sov. J. Low Temp. Phys. **4**, 520 (1978)].

⁶K.K. Likharev and L.A. Yakobson, Zh. Eksp. Teor. Fiz. **68**, 1150 (1975) [Sov. Phys. JETP **41**, 570 (1975)].

⁷I.O. Kulik and A.N. Omel'yanchuk, Fiz. Nizk. Temp. **4**, 296 (1978) [Sov. J. Low Temp. Phys. **4**, 142 (1978)].

⁸G.M. Eliashberg, Zh. Eksp. Teor. Fiz. **61**, 1254 (1971) [Sov. Phys. JETP **34**, 668 (1972)].

⁹L.P. Gor'kov and N.B. Kopnin, Zh. Eksp. Teor. Fiz. **64**, 356 (1973) [Sov. Phys. JETP **37**, 183 (1973)].

¹⁰A.I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73**, 299 (1977) [Sov. Phys. JETP **46**, 155 (1977)].