Resistive state of superconductors

B. I. Ivlev and N. B. Kopnin

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted 16 October 1978)
Pis'ma Zh. Eksp. Teor. Fiz. **28**, No. 10, 640–644 (20 November 1978)

The resistive state of a quasi-one-dimensional superconductor is treated as the result of the onset of topological singularities. The current–voltage characteristic is calculated on this basis.

PACS numbers: 74.30. – e

It is well known that in an infinite superconductor with a homogeneous and stationary order parameter there can exist no constant and homogeneous electric field, for otherwise the potentials that increase linearly with time or with the coordinate would destroy the superconductivity. In a real situation, a state, customarily called resistive, is established in a superconducting sample to whose ends a potential difference is applied. This state is characterized by destruction of the macroscopically coherent state in certain points of the sample, where jumps of the order parameter take place. The concept of phase-slippage centers in a superconductor was introduced by Langer and Ambegaokar.[1]

A resistive state can be realized in a superconductor by several methods. One of them is the case of static slippage centers, where the chemical potential of the pairs becomes discontinuous at each of the centers, so that the condensate adjusts itself to the increasing chemical potential of the quasiparticles. This picture is quasistationary if the Josephson current through each center is disregarded. It was investigated in a number of papers.[2–4] It was treated most consistently by Galaiko.[5]

The other method is connected with the case of dynamic phase-slippage centers. The order parameter vanishes in this case at certain instants of time and at certain points of the superconductor. This is precisely the situation indicated in Ref. 1 and investigated also by Rieger, Scalapino, and Mercereau.[6]

Let us dwell in greater detail on this case. Assume we have an infinite narrow superconducting sample. The complex order parameter is $\Delta = |\Delta| \exp(-2ie\chi/c\hbar)$, where $-2e\chi/c\hbar$ is the phase. In addition to the electromagnetic potentials $A = A_x$ and ϕ, we can introduce the gauge-invariant combinations $Q = A + \partial\chi/\partial x$ and $\Phi = \phi - \partial\chi/\partial(ct)$. We introduce in the two-dimensional space $\rho = \{x;\ ct\}$ the two-dimensional vectors

$$\mathbf{q} = \{Q;\ -\Phi\} \qquad \mathbf{a} = \{A; -\phi\}, \tag{1}$$

which are related by

$$\mathbf{q} = \mathbf{a} + \partial\chi/\partial\vec{\rho}. \tag{2}$$

Then, if the circulation of the vector $\mathbf{q}$ vanishes along a certain closed contour L

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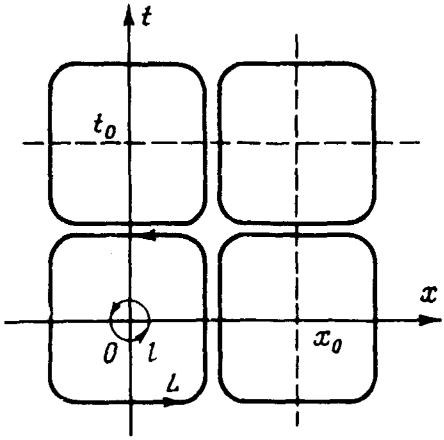

FIG. 1.

enclosing the area S (see Fig. 1) then

$$\oint \mathbf{q}\, d\rho = \int_S \mathrm{rot}\,\mathbf{a}\, d\mathbf{S} + \oint_L \frac{\partial \chi}{\partial \vec{\rho}}\, \partial\vec{\rho} = 0; \quad |d\mathbf{S}| = d^2\rho. \tag{3}$$

Since the change of the phase on going around the closed contour should be a multiple of 2π, and the electric field is formally equal to the vector

$$E = -\frac{1}{c}\frac{\partial A}{\partial t} - \frac{\partial \phi}{\partial x} = (\mathrm{rot}\ \mathbf{a})_3, \tag{4}$$

which is directed along $d\mathbf{S}$, it follows that

$$\int_S E d^2\rho = n\Phi_0, \tag{5}$$

where $\Phi_0 = \pi c\hbar/e$ is the "flux quantum" (in analogy with vortices in type-II superconductors[7]).

Thus, the phase slippage centers can be visualized as topological singularities similar to vortices in two-dimensional space-time. If they form a regular lattice, then the contour L can be taken to be the boundary of the cell, and by virtue of the periodicity the circulation of the vector **q** is equal to zero. For the case of a rectangular lattice of solitons, the electric field averaged over the coordinate and over the time, which is in fact the quantity measured in the experiment, takes the form

$$\bar{E} = \frac{1}{x_0 t_0 c}\int_S E d^2\rho = \frac{n\Phi_0}{c x_0 t_0}. \tag{6}$$

This relation, together with the boundary condition for an infinitesimally small contour l (see Fig. 1),

$$\int_l \mathbf{q}\, d\vec{\rho} = -n\Phi_0, \tag{7}$$

is useful for the determination of the current–voltage characteristic of the superconductor, if the periods of the oscillations are determined from the given current.

We consider a superconductor with large concentration of paramagnetic impurities.[8] In terms of the units $\widetilde{\Delta}^2 = 2\pi^2(T_c^2 - T^2)$; $\xi^2 = 6D/r_s\widetilde{\Delta}^2$; $\tilde{t}^{-1} = 2\tau_s\widetilde{\Delta}^2$; $2e\widetilde{\Phi} = \tilde{t}^{-1}$, $(2e/c)\widetilde{Q} = \xi^{-1}(\hbar = 1)$, the equations for the modulus of the order parameter and for the gauge-invariant potentials take the form

$$12\frac{\partial|\Delta|}{\partial t} - \frac{\partial^2|\Delta|}{\partial x^2} + (|\Delta|^2 - 1)|\Delta| + Q^2|\Delta| = 0, \quad -\left(|\Delta|^2 Q + \frac{\partial Q}{\partial t} + \frac{\partial \Phi}{\partial x}\right) = j;$$

$$12\,\Phi|\Delta|^2 = -\frac{\partial}{\partial x}(|\Delta|^2 Q). \tag{8}$$

Kramer and Baratoff[9] found by numerical methods that at $j > j_{\min} \approx 0.74 j_c$, where $j_c = 2/3(3)^{1/2}$ is the Ginzburg–Landau critical current, there exists a time-periodic solution of the type considered above. Furthermore, if $j - j_{min} \to +0$, then the temporal period tends to infinity, and the solution for the order parameter as $t \to \pm\infty$ goes over into the unstable static solution of Langer and Ambegaokar.[1]

$$\Delta_\infty(x) = \Delta_0 - (3\Delta_0^2 - 2)\,\mathrm{ch}^{-2}\left[x\left(\frac{3\Delta_0^2 - 2}{2}\right)^{1/2}\right]. \tag{9}$$

The symbol "∞" labels the coordinate period of the structure. In the numerical calculation of Ref. 9, boundary conditions could occur periodically in time for only one phase-slippage center over the length of the sample. For an infinite sample, however, or under boundary conditions that admit of the onset of new solitons in the sample, their number can vary in space. For this reason, the structure of the period, both with respect to coordinate and with respect to time, is determined by the supercriticality $j - j_{min}$.

At $j = j_{min}$, the solution obtained in Ref. 9 for (8) is $\Delta(x,t) = \Delta_\infty(x) + \psi(x,t)$, where $\psi \to 0$ as $x \to \pm\infty$ and $t \to \pm\infty$. It can be verified by analyzing the curve of Ref. 9 for the chemical potential that the quantization rule (5) with $n = 1$ is satisfied in this case. At $(j - j_{\min}) \ll j_{\min}$ we can write the solution in the form

$$\Delta(x,t) = \Delta_{x_0}(x) + \sum_{nm}\psi(x - nx_0,\ t - mt_0) + \Delta_1(x,t), \tag{10}$$

where $\Delta x_0(x)$ is the static solution of Eqs. (8) with period x_0 at the current $j = j_{\min}$; at $x \ll x_0$ this solution differs little from (9). Then, at sufficiently large periods, the function Δ_1 is small. The right-hand side of the equation for Δ_1 contains, besides $(j - j_{\min})$, small overlap terms of the type $\psi(x,t)\psi(x,t + t_0) \sim \exp(-bt_0)$ and in addition $[\Delta_{x_0}(x) - \Delta_\infty(x)] \sim \exp(-ax_0)$. It can be assumed that the exact solution of Eqs. (8) should be arranged in such a way that all these quantities are of the same order. From this we get for the periods of the soliton lattice

$$x_0 \sim \xi \ln\frac{j_{min}}{j - j_{min}} \qquad t_0 \sim \tilde{t}\ln\frac{j_{min}}{j - j_{min}}. \tag{11}$$

Using now relation (6) for the average electric field, we obtain the current-voltage characteristic at $(j - j_{\min}) \ll j_{\min}$

$$\frac{j - j_{min}}{j_{min}} \sim \exp\left[-C\left(\frac{j_{min}}{\sigma E}\right)^{1/2}\right], \qquad C \sim 1. \tag{12}$$

We note in conclusion that the considered periodic structure can in principle be reasonsible for the experimentally observed electromagnetic radiation from a superconducting sample in the resistive state.[10] In addition, the entrance of new solitons into the sample through its boundary should lead to voltage jumps, which were observed experimentally in Ref. 11.

We are grateful to A.I. Larkin and Yu. N. Ovchinnikov, as well as to E.B. Bogomol'nyĭ and S.V. Manakov, for valuable discussions.

[1]J.S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498 (1967).
[2]V.P. Galaĭko, Zh. Eksp. Teor. Fiz. **66**, 379 (1974) [Sov. Phys. JETP **39**, 181 (1974)].
[3]W.J. Skocpol, M.R. Beasley, and M. Tinkham, J. Low Temp. Phys. **16**, 145 (1974).
[4]H.J. Fink and R.S. Poulsen, Phys. Rev. Lett. **32**, 762 (1974); Phys. Rev. B **11**, 1870 (1975).
[5]V.P. Galaĭko, Zh. Eksp. Teor. Fiz. **71**, 273 (1976) [Sov. Phys. JETP **44**, 141 (1976)].
[6]T.J. Rieger, D.J. Scalapino, and J.E. Mercereau, Phys. Rev. B **6**, 1734 (1972).
[7]D. Saint-James *et al.*, Type II Superconductivity, Pergamon, 1970.
[8]L.P. Gor'kov and G.M. Eliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
[9]L. Kramer and A. Baratoff, Phys. Rev. Lett. **38**, 518 (1977).
[10]G.E. Churilov, V.M. Dmitriev, and A.P. Beskorsyĭ, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 231 (1969) [JETP Lett. **10**, 146 (1969)].
[11]J.D. Meyer, Appl. Phys. **2**, 303 (1973).