and for the baryon decuplet

$$N^{*++} - N^{*+} = m_1 + 4m_2 + 4m_3, \quad Y_1^{*+} - Y_1^{*0} = m_1 + m_2 + m_3,$$
$$N^{*+} - N^{*0} = m_1 + m_2 + m_3, \quad Y_1^{*0} - Y_1^{*0} = m_1 - 2m_2 - 2m_3, \qquad (3)$$
$$N^{*0} - N^{*-} = m_1 - 2m_2 - 2m_3, \quad \Xi^{*0} - \Xi^{*-} = m_1 - 2m_2 - 2m_3$$

From (2) and (3) follow all the relations for the mass differences obtained in SU(3) symmetry.[3] In addition, the following relations hold between the octet and decuplet particle masses:

$$N^{*+} - N^{*0} = Y_1^{*+} - Y_1^{*0} = p - n$$
$$N^{*0} - N^{*-} = Y_1^{*0} - Y_1^{*-} = \Xi^{*0} - \Xi^{*-} = p - n - \Sigma^+ - \Sigma^- + 2 \qquad (4)$$
$$N^{*++} - N^{*+} = p - n + \Sigma^+ + \Sigma^- - 2$$

Comparison of (4) with the experimental mass values is impossible at present, owing to the large errors in the determination of the resonance masses.

In conclusion, the author thanks A. I. Akhiezer for discussing the present results.

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1) We note that the term of (1) linear in C_A^A, is the result of the relations $\underset{\rightarrow}{C}_A^B \underset{\rightarrow}{C}_B^C = 3C_A^B C_B^C = C_A^C + (2/3)\delta_A^C$.

ELECTRIC RESISTIVITY OF METALS WITH LOW MAGNETIC IMPURITY IN THE PRESENCE OF IMPURITY FERROMAGNETISM AND AN EXTERNAL MAGNETIC FIELD

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Original submitted 1 March, 1965

In an earlier note[1] we presented the results of calculations of the electric resistivity of a nonmagnetic metal with small admixture of magnetic atoms. In the present paper we consider the case of impurity ferromagnetism, and determine the dependence of the resistivity on an external magnetic field.

We used the same spin diagram technique as in [1]. The ferromagnetism and the external magnetic field were taken into account in the sense of [2], i.e., in the form of some effective field acting on the electrons and impurity "particles." Thus, in place of the usual "free" Green's functions, the functions used in this "field" were

$$G = \frac{1}{i - \xi + \sigma_z P} \qquad g = \frac{1}{i - \lambda + S_z Q} \tag{1}$$

The values of P and Q were determined in a self-consistent manner, so that the following equations were obtained (see also [2]):

$$\begin{aligned} P &= \mu_0 H + JcSB_S\left(\frac{S\,Q}{T}\right) \\ Q &= g\mu_0 H + \frac{3zJ}{2\varepsilon_F} P \end{aligned} \tag{2}$$

where

$$B_S(x) = \frac{2S+1}{2S}\coth\frac{(2S+1)x}{2S} - \frac{1}{2S}\coth\frac{x}{2S}$$

is the Brillouin function, c the atomic concentration, z the number of electtrons per atom, and $g \approx 2$ is the gyromagnetic ratio for the impurity. In addition to changing the Green's functions, the presence of the "field" changes the normalization factor when averaging over the impurity spins, from $e^{\lambda/T}/(2S + 1)$ to

$$e^{\lambda/T}\sinh\frac{Q}{2T} \Big/ \sinh\frac{Q(2S+1)}{2T}$$

As before, the calculation was made with logarithmic accuracy, i.e., it was assumed that $(J/\varepsilon_F)\ln(\varepsilon_F/Q)$ are arbitrary, but $J/\varepsilon_F \ll 1$. It turned out that the change in the electron energy in the field (the quantity P) does not enter in the result explicitly, although, in accord with (2), it does determine the value of Q.

In the case when the ordinary resistivity ρ_{ord}, connected with the nonexchange interaction vetween the electron and the impurity, greatly exceeds ρ_{ex}, due to the exchange interaction, the result takes the form

$$= \rho_{ord} + \rho_{0,ex}\left[1 - B_S\left(\frac{SQ}{T}\right)\frac{\sinh Q/T - Q/T}{\cosh Q/T - 1}\right]\left(1 + \frac{3Jz}{2\ \ _F}\ln\frac{\varepsilon_F}{\max(Q,T)}\right)^{-2} \tag{3}$$

where $\rho_{0,ex} = [3\pi^2 mJ^2 S(S+1)c]/2N\varepsilon_F e^2\hbar$ is the atomic concentration. The

limiting cases are:

$$\rho = \rho_{ord} + \rho_{0,ex}[1 - (\frac{Q}{3T})^2](1 + \frac{3Jz}{2\varepsilon_F} \ln \frac{\varepsilon_F}{T})^{-2}, \qquad Q/T << 1;$$
$$\rho = \rho_{ord} + \rho_{0,ex}[\frac{S}{S+1} + \frac{e^{-Q/T}}{S+1}(\frac{1}{S} - 2 + \frac{2Q}{T})](1 + \frac{3Jz}{2\varepsilon_F} \ln \frac{\varepsilon_F}{Q})^{-2}, \quad e^{-Q/T} << T \tag{4}$$

According to (2), in the absence of a magnetic field Q is given by

$$Q = \frac{3zJ^2cS}{2\varepsilon_F} B_S (\frac{SQ}{T}) \tag{5}$$

In particular,

$$Q = J[\frac{15}{2} \frac{czS(S+1)}{\varepsilon_F(S + S + 1/2)} (T_C - T)]^{1/2}, \qquad T_C - T << T_C;$$
$$Q \quad \frac{3czJ^2S}{2\varepsilon_F}, \qquad T << T_C \tag{6}$$

where T_C is the Curie temperature, equal to

$$T_C = [J^2czS(S + 1)]/2\varepsilon_F \tag{7}$$

This means that when $T_C = T$ we obtain $QS << T_C$, and when $T \lesssim T_C$ we have $QS \sim T_C$. On the other hand, if $g\mu_0 H >> T_C$, then

$$Q \quad g\mu_0 H \tag{8}$$

It follows therefore that when H = 0 the denominator in (3) contains $\ln(\varepsilon_F/T)$ if $T >> T_C$ and $\ln(\varepsilon_F/T_C)$ if $T \lesssim T_C$. As to the numerator, it decreases when the temperature drops below T_C, with S(S + 1) replaced, in accord with (4) by S^2. The cause of this effect is very simple. The effective scattering cross section contains $(\vec{\sigma}\cdot\vec{S})(\vec{\sigma}\cdot\vec{S})$. If the spins of the impurity are not oriented, then the mean value of this product is S(S + 1). If they are fully oriented, the product becomes $\sigma_z S \cdot \sigma_z S = S^2$. Thus, ordering of the spins leads to a decrease in the numerator of (3). In the presence of a magnetic field $g\mu_0 H >> T_C$ the quantity $g\mu_0 H$ plays the same role as T_C in the absence of the field. The general form of the $\rho(T)$ curve depends on the relations between the parameters.

If J > 0, the $\rho(T)$ will decrease as the temperature drops to $T \sim T_0$, at which $Q \sim T$ (according to the foregoing, $T_0 \sim \max(T_C, g\mu_0 H)$). Beyond this, $\rho(T)$ becomes independent of the temperature.

On the other hand, if J < 0, different cases are possible. As is known from [1], in the absence of spin ordering the $\rho(T)$ curve has a mini-

mum at some value T_{min}. It results from the addition of the ordinary resistivity, which decreases with the temperature, and the increasing volume resistivity. At a temperature $T_{max} \ll T_{min}$ there is a maximum connected with the resonance in the exchange scattering of the electrons. The temperature T_{max} is independent of the electron concentration

Assume now that ordering takes place and the corresponding temperature is $T_0 \ll T_{max}$. ρ_{ex} no longer drops to zero below T_{max}, and becomes constant below T_0. If $T_{max} \ll T_0 \ll T_{min}$, then the maximum due to the resonance disappears, but a new one appears. As the temperature drops from T_{min} to T_0, the resistivity increases, and then begins to decrease because of the transition from the coefficient $S(S + 1)$ to S^2. The position of the maximum of T_0 at $T_C \ll g\mu_0 H$ corresponds to $T_0 \sim T_C$, i.e., it is approximately linear in the impurity concentration, and when $T_C \ll g\mu_0 H$ it corresponds to $T_0 \sim g\mu_0 H$, i.e., it is independent of the concentration but depends approximately linearly on the field. Finally, when $T_0 \gg T_{min}$ both the maximum and the minimum on the ρ_{ex} curve disappear.

So far we have considered only the exchange part ρ_{ex} of the resistivity. However, the magnetic field influences also ρ_{ord} because the electron orbits become bent in the presence of the field. If the collision time $\tau \ll \Omega$, where $\Omega = eB/cm$ (B is the mean field in the sample), then this effect has an order of magnitude $\rho_{ord}(\Omega\tau)^2$. An estimate shows that in the presence of an external field a decrease in the exchange resistivity, due to the ferromagnetic ordering, is much larger than the increase in ρ_{ord} due to the appearance of the internal magnetic field, However, if we compare the effects of the external field on ρ_{ex} and on ρ_{ord}, then different cases are possible. If

$$\max(T, \mu_0 H) \ll \frac{\rho_{ex}}{\rho_{ord}} \frac{\hbar}{\tau} \tag{9}$$

then the increase in the ordinary resistivity will be smaller than the decrease in the exchange resistivity. Over a mean free path $\ell \sim 10^{-3}$ cm the right side of (9) equals approximately $0.1^{\circ}K$. Thus, the total resistivity can either increase or decrease with increasing magnetic field. We note that the increase of the ordinary resistivity can be suppressed by introducing

nonmagnetic impurities. In addition, it must be borne in mind that the change in the ordinary resistivity with the changing field does not depend on the temperature, whereas the change of the exchange resistivity, as shown above, is strongly temperature dependent.

In conclusion we note that all the qualitative features obtained here for the resistivity (minima and Maxima of resistivity, vanishing of the extrema in a sufficiently strong field, negative magnetoresistance) were experimentally observed (see [3]).

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