REACTION OF PRODUCTION OF TWO SLOW PIONS IN ee AND ep COLLISIONS

M.V. Terent'ev

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An experimental study of the process of production of two soft pions in ee and ep collisions is of interest, particularly in connection with the verification of current algebra and the PCAC hypothesis.

Let us consider a two-photon pion generation mechanism (see the figure). In the figure, p_1 and p_2 are the momenta of the colliding charged particles, k_1 and k_2 are the momenta of the "soft" pions ($(k_1 + k_2)^2 \sim 4\mu_\pi^2$), and q_1 and q_2 are the momenta of the virtual quanta. The possibility of separating such a mechanism and the scale of the cross sections will be discussed briefly at the end of the article. We are interested in the amplitude $A^{ab}_{\nu\mu}(k_1, k_2; q_1, q_2)$ involved in this process, of the conversion of two quanta into two mesons (ν and μ are the polarization indices of the photons, and a and b are the isotopic indices of the pions). The main statements consist in the following:

1. Assume that all the momenta in the amplitude $A^{ab}_{\nu\mu}$ are small, and generally speaking of the same order of magnitude. Then, with accuracy $\sim\mu_\pi^2/m_{char}^2$, where m_{char} is the characteristic scale of variation of the hadronic amplitudes[1)] and $A_{\nu\mu}$ is the sum of the pole diagrams and of the contact term:

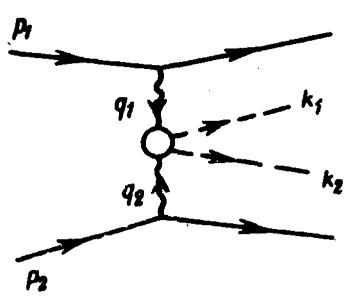
$$A_{\nu\mu} = -e^2 T_3^2 \left[2\delta_{\nu\mu} + \frac{(2p_1 - q_1)_\nu (2p_2 - q_2)_\mu}{q_1^2 - 2p_1 q_1} + \frac{(2p_2 - q_1)_\nu (2p_1 - q_2)_\mu}{q_1^2 - 2p_2 q_1} \right]. \tag{1}$$

Here $T_3^{ab} = -i\varepsilon_{3ab}$ is the isotopic matrix, $(T_3^2)^{ab} = \delta_{ab} - \delta_{a3}\delta_{b3}$, and $e^2 = 4\pi\alpha$.

2. If, as before, $k_1 \sim k_2 \sim \mu_\pi$ but $|q_1|^2 \gg \mu_\pi^2$ and $|q_2|^2 \gg \mu_\pi^2$, then obviously $q_1 \simeq -q_2 = q$, and $q \gg \mu_\pi$. In this region we have

$$A_{\nu\mu} = -e^2 T_3^2 \cdot 2(\delta_{\nu\mu} - q_\nu q_\mu / q^2) R(q^2), \tag{2'}$$

$$R(q^2) = 1 - F_\pi^{-2} \int d\kappa^2 \frac{\rho_V(\kappa^2) - \rho_A(\kappa^2)}{\kappa^2} \frac{q^2}{q^2 - \kappa^2}. \tag{2''}$$



Here F_π is the pion decay amplitude defined, say, in terms of the Goldberger-Treiman relation

$$F_\pi = \frac{m_N (G_A / G_V)}{g_r},$$

$g_r^2/4\pi = 14.6$; ρ_V and ρ_A are spectral functions of the vector and axial currents:

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1) Usually $m_{char} \sim m_N$ (nucleon mass). We assume that there is no strong cross scattering of the pions near the threshold. In addition, m_{char} can be of the order of the mass of resonance in the $\pi\pi$ scattering, if such a resonance exists.

$$\rho_Z(p^2) = -\frac{(2\pi)^3}{3}\sum_{n,\nu} <0|J_\nu^{Z,3}(0)|n><n|J_\nu^{Z,3}(0)|0>\delta(p_n - p), \tag{3}$$

$$Z = V,\ A.$$

The normalization of the currents is such that, say in the quark model,

$$J_\nu^{V,a} = \frac{1}{2}\bar\psi\gamma_\nu\tau_a\psi,\quad J_\nu^{A,a} = \frac{1}{2}\bar\psi\gamma_\nu\gamma_5\tau_a\psi.$$

The sum over the intermediate states in (3) does not include a state with a single pion. Its contribution is separated and corresponds to the first term (unity) in (2").

3. The accuracy of the representation (1) is of the order of μ^2/m^2_{char}, as already noted in Sec. 1. To derive it, we need only the conservation of the electromagnetic current and the condition $\mu/m_{char} \ll 1$. A verification of this condition, which is important for the philosophy of current algebra, would be of interest.

The accuracy of the representation (2) is of the order of μ/m_{char} - the customary accuracy of the consequences of the PCAC hypothesis[2].

To obtain (2) it is necessary to postulate conservation of the axial current as $\mu_\pi \to 0$, the usual assumptions concerning the commutators of the temporal components of the vector and axial currents, and the condition $\mu_\pi/m_{char} \ll 1$, i.e., the set of main hypotheses on which the method of current algebra is based (see [1], Ch. 1).

4. Since the matrix T_3^3 enters in (1) and (2), the amplitude for the emission of two π^0 mesons is equal to zero. The accuracy of this hindrance is determined naturally by the conditions for the applicability of these formulas.

5. The quantity $\rho_V(\kappa^2)/\kappa^2$ entering in (2") is expressed in terms of the total cross section for e^+e^- annihilation into hadrons with additional emission of two soft pions (see [2]), and the quantity $\rho_A(\kappa^2)/\kappa^2$ is proportional to the cross section of the same process but with additional emission of only one soft pion (see [2]).

If the sum rule

$$\int\rho_V(\kappa^2)\frac{d\kappa^2}{\kappa^2} = F_\pi^2 + \int\rho_A(\kappa^2)\frac{d\kappa^2}{\kappa^2}$$

is satisfied (Weinberg's second sum rule [3]), then $R(q^2)$ decreases with increasing q^2, and if the sum rule

[2] The possibility of using (2) when $|q^2| \gg m_N^2$ is apparently connected with a number of additional assumptions concerning the character of the dependence of $A_{\nu\mu}$ on the invariants qk_1 and qk_2. If the amplitude depends on these invariants only in the combination $(q - k_1)^2$ or $(q - k_2)^2$, then the representation (2) is valid for all q^2.

$$\int \rho_V(\kappa^2)\, d\kappa^2 = \int \rho_A(\kappa^2)\, d\kappa^2,$$

which is connected with the field-algebra hypothesis (Weinberg's first sum rule [3]) is satisfied, then $R(q^2)$ in (2") decreases more rapidly than q^{-2}.

If the integral in (2") is saturated by the contribution of the vector mesons (this, generally speaking, is not obligatory, but it is known that such integrals are well approximated by the contribution of the ρ and A_1 mesons, see, e.g., [3, 4]), then we must put: $\rho_V = g_\rho^2 \delta(\kappa^2 - m_\rho^2)$ and $\rho_A = g_A^2 \delta(\kappa^2 - m_A^2)$, and in addition $m_A^2 = 2m_\rho^2$ and $g_A^2 = g_\rho^2 = 2F_\pi^2 m_\rho^2$, where m_ρ is the ρ-meson mass. Here $R(q^2) = 2m_\rho^4/(q^2 - m_\rho^2)(q^2 - 2m_\rho^2)$.

6. Relation (1) can be verified by colliding-beam experiments, ep scattering, and coherent scattering of electrons by nuclei (if we choose specially kinematics corresponding to $q_2^2 \to 0$). The values of the kinematic variables in the region where the representation (1) is valid correspond precisely to the largest contribution to the total cross section. Therefore the magnitude of the effect can be readily estimated from the formulas of [5]. For 3.5×3.5 GeV^2 colliding ee beams, the cross section is $\sim 10^{-34}$ cm^2.

Relation (2) can be verified only in experiments with colliding lepton beams. At an energy 3.5×3.5 GeV^2 and $q^2 \sim 0.5$ GeV^2 (i.e., when $s \gg q^2$), the cross section can be estimated from the formula

$$d\sigma \sim \frac{16\alpha^4}{\pi} \frac{R^2(q^2)\, dq^2}{q^6} \frac{}{q^2} \sqrt{\omega_1^2 - \mu_\pi^2}\, \sqrt{\omega_2^2 - \mu_\pi^2}\, d\omega_1 d\omega_2,$$

where ω_1 and ω_2 are the pion energies. If we assume $dq^2 \sim q^2$, $d\omega_j \sim \omega_j \sim \mu_\pi$, $j = 1$ and 2, and approximate the function $R(q^2)$ by the contribution of the vector mesons (see Sec. 5), then the cross section turns out to be $\sim 10^{-38}$ cm^2.

We hope to present in a separate paper an investigation of the processes $\gamma\gamma \to 3\pi$ and $\gamma\gamma \to n\pi$ for soft pions, a discussion of the details of the derivation of formulas (1) and (2), and also a more detailed discussion of the experimental observation possibilities.

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INFLUENCE OF CRYSTAL LATTICE ON NUCLEAR PROPERTIES OF SUPERDENSE MATTER

V.A. Volodin and D.A. Kirzhnits
Physics Institute, USSR Academy of Sciences
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In connection with the discovery of pulsars and their identification with