$$\int \rho_V(\kappa^2)\,d\kappa^2 = \int \rho_A(\kappa^2)\,d\kappa^2,$$

which is connected with the field-algebra hypothesis (Weinberg's first sum rule [3]) is satisfied, then $R(q^2)$ in (2") decreases more rapidly than $q^{-2}$.

If the integral in (2") is saturated by the contribution of the vector mesons (this, generally speaking, is not obligatory, but it is known that such integrals are well approximated by the contribution of the $\rho$ and $A_1$ mesons, see, e.g., [3, 4]), then we must put: $\rho_V = g_\rho^2\delta(\kappa^2 - m_\rho^2)$ and $\rho_A = g_A^2\delta(\kappa^2 - m_A^2)$, and in addition $m_A^2 = 2m_\rho^2$ and $g_A^2 = g_\rho^2 = 2F_\pi^2 m_\rho^2$, where $m_\rho$ is the $\rho$-meson mass. Here $R(q^2) = 2m_\rho^4/(q^2 - m_\rho^2)(q^2 - 2m_\rho^2)$.

6. Relation (1) can be verified by colliding-beam experiments, ep scattering, and coherent scattering of electrons by nuclei (if we choose specially kinematics corresponding to $q_2^2 \to 0$). The values of the kinematic variables in the region where the representation (1) is valid correspond precisely to the largest contribution to the total cross section. Therefore the magnitude of the effect can be readily estimated from the formulas of [5]. For 3.5 × 3.5 $GeV^2$ colliding ee beams, the cross section is $\sim 10^{-34}$ $cm^2$.

Relation (2) can be verified only in experiments with colliding lepton beams. At an energy 3.5 × 3.5 $GeV^2$ and $q^2 \sim 0.5$ $GeV^2$ (i.e., when $s \gg q^2$), the cross section can be estimated from the formula

$$d\sigma \sim \frac{16\alpha^4}{\pi}\frac{R^2(q^2)\,dq^2}{q^6}\frac{}{q^2}\sqrt{\omega_1^2 - \mu_\pi^2}\,\sqrt{\omega_2^2 - \mu_\pi^2}\,d\omega_1 d\omega_2,$$

where $\omega_1$ and $\omega_2$ are the pion energies. If we assume $dq^2 \sim q^2$, $d\omega_j \sim \omega_j \sim \mu_\pi$, $j = 1$ and 2, and approximate the function $R(q^2)$ by the contribution of the vector mesons (see Sec. 5), then the cross section turns out to be $\sim 10^{-38}$ $cm^2$.

We hope to present in a separate paper an investigation of the processes $\gamma\gamma \to 3\pi$ and $\gamma\gamma \to n\pi$ for soft pions, a discussion of the details of the derivation of formulas (1) and (2), and also a more detailed discussion of the experimental observation possibilities.

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## INFLUENCE OF CRYSTAL LATTICE ON NUCLEAR PROPERTIES OF SUPERDENSE MATTER

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In connection with the discovery of pulsars and their identification with neutron stars, the problem of the equilibrium chemical composition of superdense matter has become very timely. In the corresponding calculations [1 - 4] it is customary to ignore completely the effects of the Coulomb interaction between the electrons and the nuclei. Yet under the conditions of the pulsar crust, the Coulomb forces are so appreciable, that they not only lead to crystallization of the matter [5], but turn out to be also large on the nuclear energy scale. In fact, the lattice Coulomb energy (abbreviated l.C.e.) per nucleus is given by

$$\mathcal{E}_L = -\frac{9}{10}\frac{Z^2 e^2}{R}, \quad R = \left(\frac{4\pi}{3} n_N\right)^{-1/3}, \tag{1}$$

where R is a quantity on the order of the distance between nuclei, $n_N$ is the concentration of the nuclei, and Z is their charge. By assuming the values $n_N \sim 10^{34}$ cm$^{-3}$ and $Z \sim 50$ (see [3]), we get $\mathcal{E}_L \sim 100$ MeV. One should therefore expect a noticeable influence of the l.C.e. on the nuclear characteristics of superdense matter[1].

This effect, tending to increase the equilibrium charge (and the mass number) of the nucleus and to stabilize the nuclei, which are unstable in the isolated state against fragmentation accompanied by subdivision of the charge (fission, $\alpha$ decay). Indeed, comparing (1) with the Coulomb energy of the nucleus

$$\mathcal{E}_N^c = \beta\frac{Z^2}{A^{1/3}}, \quad \beta = \frac{3}{5}\frac{e^2}{r_0},$$

we see that allowance for the l.C.e. reduces to an effective decrease of the Coulomb repulsion in the nucleus, described by the factor

$$W = 1 - \frac{3}{2}\frac{R_0}{R} = 1 - \frac{3}{2}\left(\frac{n_N}{n_0}\right)^{1/3}, \tag{2}$$

where $R_0 = r_0 A^{1/3}$ is the radius of the nucleus. Here $n_0 = 1.3 \times 10^{38}$ cm$^{-3}$ is the concentration of the nucleons in the nucleus.

An idea of the quantitative influence of the l.C.e. on the nuclear parameters can be obtained with the aid of the model considered in [2 - 3], in which the total energy is given by

$$E = N_N(\mathcal{E}_N + \mathcal{E}_L) + N_e\mathcal{E}_e + E_G, \tag{3}$$

where $N_N$ and $N_e$ are the total numbers of the nuclei and electrons,

$$\mathcal{E}_N = M_n(A - Z) + M_p Z - \alpha A + \beta\frac{Z^2}{A^{1/3}} + \gamma A\left(1 - 2\frac{Z}{A}\right)^2 + \delta A^{2/3}$$

is the energy of the nucleus (the Weizsacker formula),

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[1] Actually, the discussed effects will apparently take place also in a liquid-like plasma, in which the short-range order Coulomb energy has a value comparable with (1).

$$\mathcal{E}_e = \frac{3}{4}\hbar c\,(3\pi^2 n_e)^{1/3}$$

is the energy of the ultrarelativistic electron gas, and $E_G$ is the energy of the neutron-proton gas.

It is convenient to minimize (3) at a constant density, with allowance for the conservation laws for the baryon number $N = AN_N + N_n + N_p$ and for the charge $N_e = ZN_N + N_p$, in terms of the variables $Z/A \equiv \xi$, $Z^2/A \equiv \eta$, $A(N_N/N) \equiv x$, and $N_p/N \equiv y$. In terms of these variables we have

$$E/N = x[-a + \beta W \xi^{2/3}\eta^{2/3} + \gamma(1-2\xi)^2 + \delta\,\xi^{2/3}\eta^{-1/3}] + \frac{3}{4}\hbar c\,(3\pi^2 n)^{1/3}(\xi x + y)^{4/3} + \frac{E_G}{N}(n|x,y), \qquad (4)$$

where $W \equiv 1 - (3/2)(n/n_0)^{1/3}x^{1/3}$ and n is the baryon concentration. It is important to note that the ratio of the l.C.e. to the electron energy, which equals $4.6 \times 10^{-3}Z^{2/3}$, is small, and therefore the difference between W and unity has practically no effect on the equation obtained by varying (4) with respect to the parameters $\xi$, x, and y, on which the electron energy depends. Therefore the equilibrium values of the indicated parameters can be taken from the calculations in which the l.C.e. is not taken into account. The only equation from which the electron energy drops out, and therefore the influence of the l.C.e. is appreciable, corresponds to variation of (4) with respect to $\eta$ and gives

$$\eta \equiv \frac{Z^2}{A} = \frac{\delta}{2\beta W} = \frac{\eta_0}{W}.$$

Hence

$$Z = \frac{Z_0}{W}, \qquad A = \frac{A_0}{W}, \qquad (5)$$

where $Z_0$, $A_0$, and $\eta_0$ are quantities calculated without allowance for the l.C.e. Using the values given in [3], namely $Z_0 = 39$ and $A_0 = 132$, corresponding to the neutronization boundary ($\rho = 2.8 \times 10^{11}$ g/cm$^3$), we obtain $W = 0.84$, $Z = 47$, and $A = 146$. At the limit of existence of nuclei ($\rho = 4.3 \times 10^{13}$ g/cm$^3$) we have $Z_0 = 51$, $A_0 = 211$, and $W = 0.72$, $Z = 71$, $A = 292$. We note that allowance for the influence of the l.C.e. on $\xi$, x, and y increases these numbers to $Z = 74$ and $A = 308$.

No matter how crude the model is (it does not take into account the interaction of neutron-proton gas with the nuclei, shell effects, etc.), it reinforces our qualitative ideas concerning the possible existence of nuclei with anomalously large values of Z and A in superdense matter. In any case, our results point to the need for taking into account Coulomb effects of the lattice when considering the chemical composition and the equation of state of superdense matter and the equilibrium configurations of neutron stars.

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## EXTENSION OF THE ALGEBRA OF POINCARE GROUP GENERATORS AND VIOLATION OF P INVARIANCE

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One of the main requirements imposed on quantum field theory is invariance of the theory to the Poincare group [1]. However, only a fraction of the interactions satisfying this requirement is realized in nature. It is possible that these interactions, unlike others, have a higher degree of symmetry. It is therefore of interest to study different algebras and groups, the invariance with respect to which imposes limitations on the form of the elementary particle interaction. In the present paper we propose, in constructing the Hamiltonian formulation of the quantum field theory, to use as the basis a special algebra $\mathcal{R}$, which is an extension of the algebra $\mathcal{P}$ of the Poincare group generators. The purpose of the paper is to find such a realization of the algebra $\mathcal{R}$, in which the Hamiltonian operator describes the interaction of quantized fields.

The extension of the algebra $\mathcal{P}$ is carried out in the following manner: we add to the generators $P_\mu$ and $M_{\mu\nu}$ the bispinor generators $W_\alpha$ and $\overline{W}_\beta$, which we shall call the generators of spinor translations. In order to obtain the algebra $\mathcal{R}$, it is necessary to find the Lorentz-invariant form of the permutation relations between the translation generators. In order not to violate subsequently the connection between the spin and statistics, we shall consider anticommutators of the operators $W_\alpha$ and $\overline{W}_\beta$. A generalization of the Jacobi identities imposes stringent limitations on the form of the possible commutation relations between the algebra operators. We confine ourselves to consideration of only those algebras $\mathcal{R}$, in which there are no subalgebras Q such that $\mathcal{P} \subset Q$ and $\mathcal{P} \neq Q$. This choice is governed by the fact that the remaining algebras $\mathcal{R}$ are obtained by further extending the algebras $\mathcal{R}$, and the field theories corresponding to them will have a still higher degree of symmetry.

An investigation of the albegras $\mathcal{R}$ has shown that upon spatial inversion they do not go over into themselves for any choice of the structure constants of the algebra. As a result, in a field theory that is invariant against such an algebra, the parity should not be conserved[1]), and the form of the nonconservation is completely determined by the algebra itself. We shall stop to discuss one of the algebras $\mathcal{R}$:

$$[M_{\mu\nu}, M_{\sigma\lambda}]_- = i(\delta_{\mu\sigma}M_{\nu\lambda} + \delta_{\nu\lambda}M_{\mu\sigma} - \delta_{\mu\lambda}M_{\nu\sigma} - \delta_{\nu\sigma}M_{\mu\lambda}); \quad [P_\mu, P_\nu]_- = 0;$$

$$[M_{\mu\nu}, P_\lambda]_- = i(\delta_{\mu\lambda}P_\nu - \delta_{\nu\lambda}P_\mu); \quad [M_{\mu\nu}, W]_- = \frac{i}{4}[\gamma_\mu, \gamma_\nu]_- W; \quad \overline{W} = W^+\gamma_0. \tag{1a}$$

$$[W_\pm, \overline{W}]_+ = \overset{+}{\gamma}_\mu P_\mu; \quad [W, W]_+ = 0; \quad [P_\mu, W]_- = 0, \tag{1b}$$

1)A more detailed analysis of this question will be the subject of a separate paper.