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# EXTENSION OF THE ALGEBRA OF POINCARE GROUP GENERATORS AND VIOLATION OF P INVARIANCE

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Submitted 10 March 1971
ZhETF Pis. Red. 13, No. 8, 452 - 455 (20 April 1971)

One of the main requirements imposed on quantum field theory is invariance of the theory to the Poincare group [1]. However, only a fraction of the interactions satisfying this requirement is realized in nature. It is possible that these interactions, unlike others, have a higher degree of symmetry. It is therefore of interest to study different algebras and groups, the invariance with respect to which imposes limitations on the form of the elementary particle interaction. In the present paper we propose, in constructing the Hamiltonian formulation of the quantum field theory, to use as the basis a special algebra $\mathcal{R}$, which is an extension of the algebra $\mathcal{P}$ of the Poincare group generators. The purpose of the paper is to find such a realization of the algebra $\mathcal{R}$, in which the Hamiltonian operator describes the interaction of quantized fields.

The extension of the algebra $\mathcal{P}$ is carried out in the following manner: we add to the generators $P_\mu$ and $M_{\mu\nu}$ the bispinor generators $W_\alpha$ and $\bar{W}_\beta$, which we shall call the generators of spinor translations. In order to obtain the algebra $\mathcal{R}$, it is necessary to find the Lorentz-invariant form of the permutation relations between the translation generators. In order not to violate subsequently the connection between the spin and statistics, we shall consider anticommutators of the operators $W_\alpha$ and $\bar{W}_\beta$. A generalization of the Jacobi identities imposes stringent limitations on the form of the possible commutation relations between the algebra operators. We confine ourselves to consideration of only those algebras $\mathcal{R}$, in which there are no subalgebras Q such that $\mathcal{P} \subset Q$ and $\mathcal{P} \neq Q$. This choice is governed by the fact that the remaining algebras $\mathcal{R}$ are obtained by further extending the algebras $\mathcal{R}$, and the field theories corresponding to them will have a still higher degree of symmetry.

An investigation of the albegras $\mathcal{R}$ has shown that upon spatial inversion they do not go over into themselves for any choice of the structure constants of the algebra. As a result, in a field theory that is invariant against such an algebra, the parity should not be conserved[1)], and the form of the nonconservation is completely determined by the algebra itself. We shall stop to discuss one of the algebras $\mathcal{R}$:

$$[M_{\mu\nu}, M_{\sigma\lambda}]_- = i(\delta_{\mu\sigma}M_{\nu\lambda} + \delta_{\nu\lambda}M_{\mu\sigma} - \delta_{\mu\lambda}M_{\nu\sigma} - \delta_{\nu\sigma}M_{\mu\lambda}); \quad [P_\mu, P_\nu]_- = 0; \tag{1a}$$

$$[M_{\mu\nu}, P_\lambda]_- = i(\delta_{\mu\lambda}P_\nu - \delta_{\nu\lambda}P_\mu); \quad [M_{\mu\nu}, W]_- = \frac{i}{4}[\gamma_\mu, \gamma_\nu]_- W; \quad \bar{W} = W^+\gamma_0.$$

$$[W_\pm, \bar{W}]_+ = \gamma_\mu^+ P_\mu; \quad [W, W]_+ = 0; \quad [P_\mu, W]_- = 0, \tag{1b}$$

[1)] A more detailed analysis of this question will be the subject of a separate paper.

where $\overset{\pm}{\gamma}_\mu = \overset{\pm}{s}\gamma_\mu$; $\overset{\pm}{s} = (1/2)(1 \pm \gamma_5)$; $\gamma_5^2 = 1$, the spinor indices have been omitted, and an expression of the type $d_\mu d_\mu$ is defined here and below as $d_0 d_0 - d_1 d_1 - d_2 d_2 - d_3 d_3$.

The irreducible representations of the algebra (1) can be reduced in terms of the irreducible representations of its subalgebra $\mathfrak{P}$.[1] A case of physical interest is one in which the reduction is carried out in accordance with the representations $\mathfrak{P}$ characterized by a mass and spin. In the simplest irreducible representation of this type, the operator $W^0$ (the index "0" denotes henceforth that the operator is bilinear in the field operators) has the following form:

$$W^0 = \overset{+}{s} W^0 = \frac{1}{i} \int d^3x \left( \phi^*(x) \overset{\leftrightarrow}{\partial}_0 \overset{+}{s} \psi_1(x) + \omega(x) \overset{\leftrightarrow}{\partial}_0 \overset{+}{s} \psi_1^c(x) \right), \tag{2}$$

where $\phi(x)$ and $\omega(x)$ are scalar Hermitian fields, $\psi_1(x)$ is a spinor field, and the index "c" denotes charge conjugation. The mass of all the fields is the same and equals m. The operators $P_\mu$ and $M_{\mu\nu}$ have the same usual form they have in the absence of interaction. Another irreducible representation contains the Hermitian scalar field $\chi(x)$ and the transverse Hermitian vector field $A_\mu(x)$, and also the spinor field $\psi_2(x)$. The mass $\mu$ of these fields is also the same. The generator of the spinor translations is written in this representation as follows:

$$W^0 = \overset{+}{s} W^0 = \frac{1}{i\sqrt{2}} \int d^3x \left( \chi(x) \overset{\leftrightarrow}{\partial}_0 \overset{+}{s} \psi_2(x) + A_\mu(x) \overset{\leftrightarrow}{\partial}_0 \overset{+}{\gamma}_\mu \psi_2(x) \right). \tag{3}$$

To verify the validity of formulas (2) and (3) it suffices to substitute them in the algebra (1) and carry out the appropriate calculations, during the course of which it is necessary to use the equations of motion and the commutation relations for the free fields.

We proceed now to describe the interaction of the field on the basis of the algebra (1). As seen from (1b), the operator $P_\mu$ not only commutes with W, but stands also in the right side of the commutation relations. Therefore if the operator W is equal to $W^0$, i.e., if it is bilinear in the field operators, then the operator $P_\mu = P_\mu^0$ is also bilinear in the field operators. In other words, the operators $W^\pm = W - W^0$ and $H^I = H - H^0$ ($H = P_0$) either vanish simultaneously (and we obtain the theory of free fields), or else do not vanish (and we deal with the theory of interacting fields). Thus, in describing the interaction of the fields it is necessary to use representations (1) such that the operator $W^I$, like the operator $H^I$, is expressed in terms of a product of fields of higher degree than the operators $W^0$ and $H^0$. Such representations (1) are best sought by using perturbation theory in the interaction representation. To this end we expand the operators $W^0 + W^I(t)$ and $H^0 + H^I(t)$ in powers of the coupling constant g:

$$\begin{aligned} W^0 + W^I(t) &= W^0 + g W_1(t) + g^2 W_2(t) + g^3 W_3(t) + \dots \quad ; \\ H^0 + H^I(t) &= H^0 + g H_1(t) + g^2 H_2(t) + g^3 H_3(t) + \dots \quad , \end{aligned} \tag{4}$$

and substitute (4) in the last relation of (1b):

$$
\begin{aligned}
&[W^0, H_1(t)]_- + [W_1(t), H^0]_- = 0;\\
&[W^0, H_2(t)]_- + [W_1(t), H_1(t)]_+ + [W_2(t), H^0]_- = 0;\\
&\cdots\cdots\cdots\cdots
\end{aligned}
\tag{5}
$$

Solving the i-th equation of (5) with respect to the operator $W_i(t)$, we obtain

$$
\begin{aligned}
&W_1(t) = i\int_{-\infty}^{t}[W^0, H_1(r)]_- dr;\\
&W_2(t) = i\int_{-\infty}^{t}([W^0, H_2(r)]_- + [W_1(r), H_1(r)]_-)dr;\\
&\cdots\cdots\cdots\cdots
\end{aligned}
\tag{6}
$$

The increments in the orbital part of the operator $M_{\mu\nu}$ can be calculated in similar fashion. By direct substitution of (6) in (1) it is easy to verify that relations (6) determine the representations (1) for an arbitrary interaction Hamiltonian. However, at an arbitrary local density of the Hamiltonian $H^I(t)$ the density of the operator $W^I(t)$ will generally speaking not be local, owing to the integration in (6) with respect to the time. Therefore, in order to obtain physical consequences, we generalize the concepts of invariance of the theory with respect to the algebra (or group) to a case when the algebra operators themselves can depend strongly on the interaction. We shall stipulate that the spatial density of the operators of the algebra be a local function of the field operators[2]. As a result of this requirement, the integrands in (6) should be total derivatives with respect to time, and relations (6) are transformed into equations for the determination of the operators $H^I(t)$ and $W^I(t)$.

Equations (6) reduce to a system of linear homogeneous equations for constant coefficients, which are introduced as indeterminate coupling constants in the most general form of the interaction Hamiltonian. It was possible to solve this system of equations for the case when $H_1(t)$ is a product of three fields, two of which are transformed in accordance with representation (2) and the complex conjugate of (2), and the third in accordance with representation (3). The system (6) has in this case unique solutions, and only the operators $W_1(t)$, $H_1(t)$, and $H_2(t)$ differ from zero. Knowing the exact form of the Hamiltonian in the interaction representation, we can reconstruct from it the Lagrangian in the Heisenberg representation:

$$
\begin{aligned}
L(x) &= (\partial_\alpha \phi^* - igA_\alpha\phi^*)(\partial_\alpha\phi + igA_\alpha\phi) - m^2\phi^*\phi + (\partial_\alpha\omega^* - igA_\alpha\omega^*)\\
&\times(\partial_\alpha\omega + igA_\alpha\omega) - m^2\omega^*\omega + \frac{i}{2}\psi_1\gamma_\alpha\overleftrightarrow{\partial}_\alpha\psi_1 - m\bar\psi_1\psi_1 - g\psi_1\gamma_\alpha\psi_1 A_\alpha\\
&+ \frac{i}{2}\bar\psi_2\gamma_\alpha\overleftrightarrow{\partial}_\alpha\psi_2 - \mu\bar\psi_2\psi_2 - \frac{1}{2}(\partial_\beta A_\alpha)^2 + \frac{\mu^2}{2}A_\alpha A_\alpha + \frac{1}{2}(\partial_\alpha\chi)^2 - \frac{\mu^2}{2}\chi^2\\
&+ g\mu(\phi^*\phi - \omega\omega^*)\chi - \frac{g^2}{2}(\phi^*\phi - \omega^*\omega)^2 + \sqrt{2}g(\bar\psi_1\bar s\psi_2\phi + \bar\psi_2\bar s\psi_1\phi^*)\\
&- \sqrt{2}g(\psi_1^c\bar s\psi_2\omega^* + \bar\psi_2\bar s\psi_1^c\omega).
\end{aligned}
\tag{7}
$$

[2] A separate paper will be devoted to a justification of this postulate, and also to its comparison with the ordinary formulation of the requirement that the theory be invariant against a transformation group.

Thus, we have obtained a model for the interaction of quantized fields with parity nonconservation, invariant against the algebra (1)

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## POSSIBLE INSTABILITY, DUE TO TRIPLE RECOMBINATION, OF THE STATES OF A SEMICONDUCTOR ILLUMINATED BY INTENSE LIGHT

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Submitted 12 March 1971
ZhETF Pis. Red. 13, No. 8, 456 - 458 (20 April 1971)

1. There are several recent experimental investigations [1 - 3] of the processes occurring in superconductors that have absorbed laser radiation of high intensity. Vavilov and co-workers [1] have shown that at high photoexcitation levels the dominant process is the band-band (triple) or Auger recombination, and determined the coefficient C of this recombination. It turns out here that the experimental value of the coefficient C differs from that calculated in accordance with [4] for silicon by four orders of magnitude, and in addition the temperature dependence predicted in [4] was not observed experimentally.

In this paper we call attention to the fact that at high light intensities the heating of the electrons and holes due to triple recombinations becomes significant. Indeed, the recombining electron gives up an energy on the order of $\Delta E$ ($\Delta E$ is the width of the forbidden band) to the electron-hole gas, which is characterized by an effective temperature $\theta$ larger than the lattice temperature T. We shall show in addition that this heating leads to instability of the stationary state of the semiconductor.

The physical reason of the instability lies in the fact that the probability of triple recombination increases rapidly with temperature $\theta$ (like $\exp[-\Delta E/\theta]$). Therefore an increase of $\theta$ as a result of recombination heating increases the probability of triple recombination, leading in turn to further increase of the temperature and so on.

2. We confine ourselves throughout to the simplest two-band model with a quadratic dispersion and equal electron and hole masses. We shall assume that the electrons (holes) are not degenerate and take into account only triple recombination (case of large I).

Let a semiconductor be illuminated by light of intensity I and frequency $\omega$ somewhat higher than the width of the forbidden band ($\hbar\omega \gtrsim \Delta E$). Then the system is described by the following equations for the electron density n and the effective temperature $\theta$:

$$\frac{dn}{dt} = Ik - an^3\left(\frac{\Delta E}{\theta}\right)^{3/2}\exp\{-\Delta E/2\theta\}, \tag{1}$$

$$\frac{3}{2}\frac{d(n\theta)}{dt} = -\frac{\theta - T}{\tau_{ph}}\left(\frac{\theta}{T}\right)^{1/2}n + \frac{\Delta E}{2}an^3\left(\frac{\Delta E}{\theta}\right)^{3/2}\exp\{-\Delta E/2\theta\}, \tag{2}$$

which are obtained in the usual manner from the kinetic equation. In (1), the