n = 6 has a lower energy than the states with lower numbers, and should be the last to dissociate.

The dissociation of the bound states can occur as a result of screening of the Coulomb interaction by free carriers. Then the only stable states are those whose effective radius is smaller than the screening length of the Coulomb interaction. With increasing temperature, the screening length decreases as the result of the increased concentration of the free carriers. Since states with higher numbers have a larger effective radius, the screening will destroy them at a lower temperature. Thus, the observed successive dissociation of the states with increasing temperature is connected with the size of their effective radius and can be attributed to screening of the Coulomb interaction.

The authors are grateful to V.I. Perel' for a number of exceedingly useful discussions and to E.I. Balashov for preparing the samples.

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PHYSICAL STATES ON DAUGHTER TRAJECTORIES IN THE DUAL AMPLITUDE UNDER THE CONDITION $\alpha(0) = 1$

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Submitted 15 February 1971; resubmitted 22 March 1971
ZhETF Pis. Red. 13, No. 9, 507 - 511 (5 May 1971)

One of the difficulties encountered in the generalized Veneziano model [1, 2] is the presence of "ghost" states with negative norm in the spectrum of the particles arising in the dual amplitude [3, 4]. There have been grounds recently for hoping that there are no such states in the case $\alpha(0) = 1$ ($\alpha(q^2)$ is the Regge-pole trajectory) [5].

We consider in this paper the physical states on the first three daughter trajectories and show that "ghosts" remain also in this case on the third daughter trajectory at large masses ($m^2 > 22\alpha'^{-1}$).

In the generalized Veneziano model [1, 2], the scattering amplitude of N + M scalar particles has an explicitly factorized form for the pole with mass k^2, $\alpha(k^2) = j$ [3, 4]. This amplitude is given by the expression

$$\Sigma \frac{V^j_{n\{\mu_\ell\}} \bar{V}^j_{n\{\mu_\ell\}}}{j - \alpha(k^2)}, \tag{1}$$

where

$$V^j_{n\{\mu_\ell\}} \equiv \langle f | n \rangle = \langle 0 | \int \Pi dx_i \, \phi(x_i p_i) \exp\{ \Sigma P^{(n)}_\mu a^{(n)}_\mu / \sqrt{n} \} | n \rangle$$

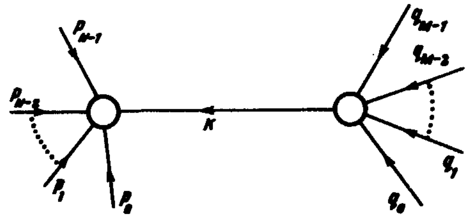
$$= \int \prod_i dx_i \, \phi(x_i, p_i) \, \Pi [P^{(\ell)}_{\mu_\ell}]^{n_\ell} \equiv \langle \Pi P^{(\ell)}_{\mu_\ell} \rangle, \tag{2}$$

and

$$| n \rangle = \Pi \frac{(a^{(\ell)+}_{\mu_\ell})^{n_\ell}}{\sqrt{n_\ell !}} | 0 \rangle$$

with

$$\Sigma n_\ell \ell \leqslant j,$$
$$\alpha(k^2) = a + \frac{1}{2}k^2. \qquad (3)$$



The remaining notation is explained in [3, 4] and in the figure.

$\bar{V}$ is the analogous expression for the vertices of the transition of the state $|n\rangle$ in M scalar particles with momenta q_j (see the figure). The sum over all states n can be rewritten in the form of a sum over the states with a given spin S:

$$\Sigma V^{j}_{n\{\mu_\ell\}} \bar{V}^{j}_{n\{\mu_\ell\}} = \sum_s \sum_p V^{js}_{p\{\mu_\ell\}} \mathcal{P}^{s\{\mu_\ell\}}_{\{\mu'_\ell\}} \bar{V}^{js}_{p\{\mu'_\ell\}}$$
$$= \sum_s \sum_p \langle f\,|\,p, j, S\rangle \langle p, j, S\,|\,f\rangle, \qquad (4)$$

where $\mathcal{P}^S$ is the propagator of the particle with spin S, and p characterizes degenerate states with j and S. Thus, the entire dual amplitude is written in the form of a sum of poles (1) over all j and $S \leq j$.

In the infinite aggregate of poles constituting the dual amplitude, one encounters states with negative residues ("ghosts") (V^{jS}_p are imaginary) [3].

Virasoro [5] observed that in this case, when a = 1, a new symmetry arises in the dual amplitude, and expressed the hope that this symmetry will lead to a complete vanishing of the "ghosts" on all trajectories.

The symmetry [5] can be expressed by the equations

$$\langle n\,|\,W_m\,|\,f\rangle = 0, \qquad (5)$$

where

$$W_m = -\Sigma\, a^{(n)+}_\mu \sqrt{n}(\sqrt{n}\,a^{(n)}_\mu - \sqrt{n+m}\,a^{(n+m)}_\mu)$$
$$-\sqrt{m}(k_\mu a^{(m)}_\mu) + (1/2)\sum_{n=1}^{m-1} \sqrt{n(n-m)}\,a^{(n)}_\mu a^{(n-m)}_\mu - \frac{1}{2}k^2 + m - 1 \qquad (5')$$

and the states $|n\rangle$ and $|f\rangle$ are defined in (2).

However, the conditions (5) with $m \geq 3$ turn out to be linear combinations of the conditions with m = 1 and 2, since

$$[W_m, W_n] = (n-m)W_{n+m} + mW_m - nW_n\,. \qquad (6)$$

If we use a system of oscillator states, then the conditions (5) interrelate vertices V^{jS}_p with identical values of j and S, and therefore (4) takes the form

$$\sum_{p_1 p_2} c_{p_1 p_2} V^{js}_{p_1\{\mu_s\}} \mathcal{P}^{s\{\mu_s\}}_{\{\mu'_s\}} \bar{V}^{js}_{p_2\{\mu'_s\}} \qquad (7)$$

where now the summation is over a smaller number of states than in (4). The task of choosing an independent system of states reduces to diagonalization of (7):

$$\sum \Gamma^{is}_{q\{\mu_s\}} \mathcal{P}^{s\{\mu_s\}}_{\{\mu'_s\}} \bar{\Gamma}^{is}_{q\{\mu'_s\}} \tag{8}$$

with

$$\Gamma^{is}_{q} = < f \mid q, j, S > = \Sigma d_{qp} V^{is}_{p} = \Sigma d_{qp} < f \mid p, j, S > \tag{9}$$

or alternately

$$\mid q, j, S > = d_{qp} \mid p, j, S > . \tag{10}$$

Let us proceed to discuss states with normal parity $(-1)^s$ on the first three daughter trajectories ($\alpha(k^2) = j$):

1. First daughter trajectory, $S = j - 1$. The condition $<n|W_1|f> = 0$ at $a = 1$ relates the vertices

$$V_1^{j,j-1} = < P^{(1)}_{\mu_1} \ldots P^{(1)}_{\mu_{j-2}} P^{(2)}_{\mu_{j-1}} > \text{ и } V_2^{j,j-1} = < P^{(1)}_{\mu_1} \ldots P^{(1)}_{\mu_{j-1}} (P^{(1)}k) >.$$

The resultant residue is equal to zero, i.e., this trajectory makes no contribution to the amplitude.

2. Second daughter trajectory, $S = j - 2$. The number of vertices $V^{j,j-2}$ is six when $j \geq 4$ (their number is 3 when $j = 2$ and 5 when $j = 3$), and the number of conditions (5) is 3 when $j \geq 3$ (2 when $j = 2$). Thus, at $j = 2$ there remains one particle. It is easy to verify that its residue is positive. At $j = 3$, the Virasoro symmetry (condition (5)) leaves two particles, but the form of (7) is degenerate (indicating a larger symmetry than given by condition (5)) and only one state arises in the sum (8). This state, like all states with odd j, drops out because the vertex V^{js} is odd with respect to the "twist" operation (in the case of odd j and a = 1). When $j \geq 4$, the matrix $\{c_{p_1p_2}\}$ (7) is singly degenerate and gives two independent states with positive residues. For $j \gg 1$, in the principal approximation in j, there is only one significant state (the vector indices of the operators $a^{(n)}_{\mu}$ will henceforth be omitted)

$$\mid 1, j, j-2 > = \frac{1}{2\sqrt{(j-1)!}} \{ a^{(3)+} a^{(1)+} - \sqrt{3} a^{(2)+} a^{(2)+} + (\sqrt{3}/2) \frac{1}{j} a^{(1)+} a^{(1)+} (a^{(1)+} a^{(1)+}) \} a^{(1)+} \ldots a^{(1)+} \mid 0 > . \tag{11}$$

3. Third daughter trajectory, $S = j - 3$. a) $j = 3$, $S = 0$ - there is no particle; b) $j = 4$, $S = 1$ - one particle with positive residue. We note that only condition (5) would lead to three independent states; c) for $j \geq 6$, there could remain five independent states (13 vertices $V^{j,j-3}$, 8 conditions (5)), but the matrix $\{c_{p_1p_2}\}$ turns out to be triply degenerate and only two independent states remain:

$$
\begin{aligned}
|1, j, j-3\rangle &= (3/2)\sqrt{23-j}\,/\sqrt{(j+20)(2j-3)(j-1)^3(j-4)!} \\
&\times\{-(4/3)(j-5)a^{(4)+}a^{(1)+}a^{(1)+} + 10-(j/\sqrt{6})(j-4)a^{(2)+}a^{(3)+}a^{(1)} \\
&+(\sqrt{2}/3)(j-4)(j-5)a^{(2)+}a^{(2)+}a^{(2)+} + (\sqrt{2}/3)j(a^{(1)+}a^{(2)+})a^{(1)+}a^{(1)+}a^{(1)} \\
&-(\sqrt{2}/4)j(a^{(1)+}a^{(1)+})a^{(2)+}a^{(1)+}a^{(1)+}\}a^{(1)+}\dots a^{(1)+}|0\rangle, \qquad (12)
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
|2, j, j-3\rangle &= [3(j-6)!(j-1)^3(j+20)]^{-1/2}\{(j+15)a^{(4)+}a^{(1)+}a^{(1)+} \\
&-(3\sqrt{3}/\sqrt{2})(j+8)a^{(2)+}a^{(3)+}a^{(1)+} + 2\sqrt{2}(j+5)a^{(2)+}a^{(2)+}a^{(2)+} \\
&+\sqrt{2}a^{(1)+}a^{(1)+}a^{(1)+}(a^{(1)+}a^{(2)+}) - (3/2\sqrt{2})(a^{(1)+}a^{(1)+})a^{(2)+}a^{(1)+}a^{(1)+}\} \\
&\times a^{(1)+}\dots a^{(1)+}|0\rangle. \qquad (13)
\end{aligned}
\tag{13}
$$

Thus, at j > 23 the state (12) is a "ghost" state, i.e., it has a negative norm. It is easy to verify that in decay into three scalar particles the states (12) and (13) are not degenerate, i.e., their vertices represent different functions of the corresponding invariants.

Some optimism may be raised by the fact that this state appears at relatively large j, at which the vertices of the transition become small[1)]. We note also that the degeneracy on this trajectory is connected with the deeper symmetry of the amplitude at a = 1 than the symmetry accounted for by the conditions (5). One more argument favoring this statement is the symmetry of the scalar four-point diagram. This amplitude contains no contributions from odd daughter trajectories, since the pole at $\alpha(s) = j$ can be represented in the form $R_j(z)/(j - \alpha(s))$, where z is the cosine of the scattering angle and

$$R_j(z) = (-1)^j R_j(-z). \tag{14}$$

It is interesting to note that (14) can be satisfied only if a = 1.

In conclusion, we are grateful to V.N. Gribov and I.T. Dyatlov for a discussion of the results.

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[1)] In this case their contribution to the amplitude is comparable with the contribution of the unitary corrections, which we assume to be small. Therefore the question of the "ghosts" must be considered jointly with the unitary corrections.

ERRATA

In the article by V.A. Kuryavtsev and E.M. Levin (Vol. 13, No. 9), an error was made in the separation of the physical state on the third daughter trajectories. Therefore formulas (12) and (13) are incorrect. The correct expressions correspond to two states that enter in the amplitude with positive residues for all $j > 5$, and therefore there are no ghosts on the entire third trajectories.

All the remaining conclusions concerning the spectrum of the states on the second and third daughter trajectories remain in force.