AMPLIFICATION OF HIGH-FREQUENCY PHONONS IN THE FIELD OF AN ELECTROMAGNETIC WAVE

E.M. Epshtein
Institute of Electronic Instruments
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It follows from the energy and momentum conservation laws that conduction electrons with a standard dispersion law $\varepsilon_p = p^2/2n$ interact only with relatively low-frequency acoustic phonons, which have $q < 2p$ (q - wave vector of the phonon; it is assumed that $p >> ms$, s is the speed of sound, $\hbar = 1$). This is manifest in a rapid decrease of the electronic damping of the phonons at $q > 2\bar{p}$ ($\bar{p}$ - characteristic value of the electron momentum) [1, 2]; under these conditions, the amplification of the sound by the carrier drift in a constant electric field decreases accordingly [3].

We show in the present paper that in the presence of an electromagnetic-wave field of frequency $\Omega >> \bar{p}^2/2m$, the situation is noticeably changed and amplification of sound with wavelength much shorter than the characteristic de Broglie wavelength of the electron, i.e., at $q >> \bar{p}$, becomes possible.

We put $q >> \bar{p}$, so that in the absence of an electromagnetic wave the electronic damping of the phonons can be neglected. If $\Omega >> \bar{p}^2/2m$, and if the amplitude of the wave field E_0 is not too large and inelastic scattering predominates, then we can confine ourselves only to processes in which one photon is absorbed (with simultaneous absorption or emission of a photon), disregard photon emission, and neglect electron heating both in the constant electric field and in the wave field (the latter is valid if $\beta \equiv e^2E_0^2/m\Omega^3 << \bar{\varepsilon}/\Omega << 1$, where $\bar{\varepsilon}$ is the characteristic energy of the electron in the absence of the wave field [4, 5]). The photon momentum is assumed to be small compared with the momenta of the electrons and phonons. Under the foregoing assumptions, the formula for the absorption coefficient of the sound by electrons in the field of the electromagnetic wave is [6]

$$\alpha(q) = \frac{\pi\Lambda^2 q}{4\rho s^2}\left(\frac{eE_0 q}{m\Omega^2}\right)^2 \sum_p n_p[\delta(\epsilon_{p+q} - \epsilon_p - \omega_q - \Omega) - \delta(\epsilon_{p-q} - \epsilon_p + \omega_q - \Omega)], \tag{1}$$

where Λ is the deformation-potential constant, ρ is the crystal density, and n_p is the electron distribution function in a constant magnetic field in the absence of an electromagnetic wave. Calculation of the absorption coefficient for a degenerate electron gas yields

$$\alpha(q) = \begin{cases} \dfrac{\Lambda^2 m^2}{4\pi\rho s}\left(\dfrac{eE_0 q}{m\Omega^2}\right)^2\left(\dfrac{q}{2} - \dfrac{m\Omega}{q}\right)\left(1 - \dfrac{qv}{qs}\right) \\ \text{for } \left|\dfrac{q}{2} - \dfrac{2m\Omega}{q}\right| < p_F, \\ 0 \text{ for } \left|\dfrac{q}{2} - \dfrac{m\Omega}{q}\right| > p_F,\ q > 2p_F, \end{cases} \tag{2}$$

where p_F is the Fermi momentum and $\vec{v}$ is the drift velocity of the electrons in a constant electric field.

It follows from (2) that when $v > s$, amplification of the phonons inside the Cerenkov cone $q \cdot v = qs$ takes place in the wave-number region $\sqrt{2m\Omega} < q \lesssim \sqrt{2m\Omega} + 2p_F$. The maximum gain in this region is smaller by an approximate factor β^{-1} than the maximum of the function $\alpha(\vec{q})$ at the point $q = 2p_F$ in the absence of the electromagnetic wave [1]. At $\Omega \sim 10^{14}$ sec^{-1} and $m \sim m_e$, the position of the gain maximum corresponds to the wave number $q \sim 10^7$ cm^{-1}, and its height at $\Lambda \sim 10$ eV, $\beta \sim 10^{-2}$, and $\varepsilon_F \sim 10^{-2}$ eV is $|\alpha| \sim 10$ cm^{-1}, so that at the temperatures needed for degeneracy the electron amplification can predominate over lattice absorption of sound.

It follows also from (2) that in the wave-number region $\sqrt{2m\Omega} - 2p_F \lesssim q < \sqrt{2m\Omega}$ there will be amplified phonons outside the Çerenkov cone (with the exception of the plane perpendicular to the vector $\vec{E}_0$). Phonon instability is possible in this region of wave numbers also in the absence of a constant field. In this case the phonon distribution is symmetrical about a plane perpendicular to $\vec{E}_0$, but anisotropic because of the presence of the factor $(\vec{E}_0 \cdot \vec{q})^2$ in (2). The latter circumstance must result in anisotropy of the kinetic coefficients.

The occurrence of electron-phonon interaction at $q \gg \bar{p}$ in the field of an electromagnetic wave is, as can be readily verified, the consequence of the conservation law $\varepsilon_{p+q} - \varepsilon_p \pm \omega_q - \Omega = 0$ for processes of interband absorption of a photon, accompanied by absorption or emission of a phonon. From the conservation laws we can also obtain the already-indicated frequency region in which the phonon instability develops. To this end it suffices, for example, to generalize the method of graphic kinematic analysis proposed in [7] to include the case when an electromagnetic field is present.

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[2] V.L. Bonch-Bruevich, Fiz. Tverd. Tela 2, 1857 (1960) [Sov. Phys.-Solid State 2, 1678 (1961)].
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In the article by E.M. Epshtein (Vol. 13, No. 9) there are misprints. Formula (2) on page 364 reads

$$\left| \frac{q}{2} - \frac{2m\Omega}{q} \right| < p_F,$$

and should read

$$\left| \frac{q}{2} - \frac{m\Omega}{2} \right| < p_F.$$

On page 365, line 7, $|\alpha| \sim 10\ \mathrm{cm}^{-1}$ should be replaced by $|\alpha| \sim 10^3\ \mathrm{cm}^{-1}$.