It follows from (2) that when $v > s$, amplification of the phonons inside the Cerenkov cone $\mathbf{q}\cdot\mathbf{v} = qs$ takes place in the wave-number region $\sqrt{2m\Omega} < q \lesssim \sqrt{2m\Omega} + 2p_F$. The maximum gain in this region is smaller by an approximate factor β^{-1} than the maximum of the function $\alpha(\vec{q})$ at the point $q = 2p_F$ in the absence of the electromagnetic wave [1]. At $\Omega \sim 10^{14}$ sec^{-1} and $m \sim m_e$, the position of the gain maximum corresponds to the wave number $q \sim 10^7$ cm^{-1}, and its height at $\Lambda \sim 10$ eV, $\beta \sim 10^{-2}$, and $\varepsilon_F \sim 10^{-2}$ eV is $|\alpha| \sim 10$ cm^{-1}, so that at the temperatures needed for degeneracy the electron amplification can predominate over lattice absorption of sound.

It follows also from (2) that in the wave-number region $\sqrt{2m\Omega} - 2p_F \lesssim q < \sqrt{2m\Omega}$ there will be amplified phonons outside the Çerenkov cone (with the exception of the plane perpendicular to the vector $\vec{E}_0$). Phonon instability is possible in this region of wave numbers also in the absence of a constant field. In this case the phonon distribution is symmetrical about a plane perpendicular to $\vec{E}_0$, but anisotropic because of the presence of the factor $(\vec{E}_0\cdot\vec{q})^2$ in (2). The latter circumstance must result in anisotropy of the kinetic coefficients.

The occurrence of electron-phonon interaction at $q \gg \bar{p}$ in the field of an electromagnetic wave is, as can be readily verified, the consequence of the conservation law $\varepsilon_{p+q} - \varepsilon_p \pm \omega_q - \Omega = 0$ for processes of interband absorption of a photon, accompanied by absorption or emission of a phonon. From the conservation laws we can also obtain the already-indicated frequency region in which the phonon instability develops. To this end it suffices, for example, to generalize the method of graphic kinematic analysis proposed in [7] to include the case when an electromagnetic field is present.

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CONCERNING ONE POSSIBILITY OF INCREASING THE CLASSICAL COEFFICIENTS FOR TRANSPORT ACROSS A STRONG MAGNETIC FIELD

L. Kovrizhnykh
P.N. Lebedev Physics Institute, USSR Academy of Sciences
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We wish to call attention in this paper to one of the mechanisms whereby the coefficients of diffusion and thermal conductivity across a strong magnetic field can be increased; this mechanism appears in the presence of intense low-frequency potential oscillations in the plasma.

It is well known [1, 2] that allowance for the toroidality leads to a modification of the old classical transport coefficients. Physically this is connected with the so-called "mixing" phenomenon [4], in other words, with the fact that the surfaces on which the drift trajectories of the particles lie differ from the magnetic surfaces resulting from the presence of the toroidal drift. If we denote by Δ_j the deviation of the drift trajectory of the electrons ($j = e$) and of the ions ($j = i$) from the magnetic surface, which we shall assume for simplicity to be in the form of a cylinder with r = const, then the coefficients of radial diffusion D and of the thermal conductivity κ_j will be of the following orders of magnitude[1] [4, 5]

$$D \approx \nu_e \Delta_e^2; \qquad \kappa_j \approx \nu_{jj} N \Delta_j^2, \tag{1}$$

where ν_e, ν_{ee}, and ν_{ii} are the effective frequencies of the electron-ion, electron-electron, and ion-ion collisions, and N is the plasma density[2].

Formulas (1) are quite obvious, and since a detailed quantitative analysis of the transport processes, based on diffusion regarded as Brownian motion and on an analysis of the drift trajectories is presented in [5], we shall not discuss them here in detail.

The displacement Δ_j in formula (1) can be estimated with the aid of the drift equations of motion. It is easily seen, however, that in the case of sufficiently small displacement $\Delta_j \ll r$, their order of magnitude is

$$\Delta_j \approx v_n^j / \Omega_j, \tag{2}$$

where v_n^j is the drift-velocity component normal to the magnetic surface, and Ω_j is the characteristic frequency of motion of the particle along the small azimuth ϕ. In the case of toroidal systems in which there are not azimuthal electric fields, v_n^j is obviously equal to the toroidal-drift velocity

$$v_n^j \approx \frac{1}{R} \frac{v_j^2}{\omega_j}, \tag{3}$$

where $v_j = \sqrt{T_j/m_j}$ is the average thermal velocity and $\omega_j = e_j B/m_j c$ is the Larmor frequency.

However, the presence of an azimuthal field also causes drift motion of the particles across the magnetic surfaces, and consequently can also lead to an increase of the transport coefficient, which is not associated directly

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[1] To be sure, it is necessary to indicate that in the case of a fully ionized plasma and at very low collision frequencies, when the free-path time becomes larger than the characteristic period of motion of the trapped particles, ν_e and ν_{ii} increase by a factor R/r compared with the ordinary effective collision frequencies (r and R are the minor and major radii of the torus), owing to the differential character of the integral of the Coulomb collisions.

[2] We note that the presence of plasma noise of sufficiently high frequency can cause the effective frequency of the electron-electron collisions ν_{ee} to be much larger than the effective frequency of the electron-ion collisions ν_e, and consequently the coefficient of electronic thermal conductivity κ_e will exceed the diffusion coefficient DN.

with the presence of toroidality [5].

Thus, let us assume that sufficiently intense low-frequency oscillations (the nature of which will not be discussed here) are excited in the plasma and are described by a potential

$$\Phi = \sum_m \Phi_m(r) e^{im\phi + i\omega_m t}, \quad m = 1, 2, 3, \ldots \tag{4}$$

and let us estimate with the aid of formulas (1) and (2) the influence of such oscillations on the diffusion and thermal conductivity of the plasma along the minor radius in magnetic traps of the "Tokamak" type, characterized by longitudinal and azimuthal magnetic-field components B_ζ and $B_\phi = \theta B_\zeta \ll B_\zeta$, respectively.

If the oscillations (4) do not have the character of noise, then in the absence of collisions they can obviously not lead to transport of particles or energy across the magnetic field, since the time averaged radial displacement Δ_j vanishes in this case. But in the presence of collisions, the situation changes, since each collision causes the particle to "forget," as it were, its prior velocity and coordinates. Naturally, this becomes most strongly manifest in the case of sufficiently low oscillation frequencies, when $\omega_m \ll \nu_j$. If in addition the characteristic frequency of the azimuthal motion Ω_j exceeds the field oscillation frequency ω_m, then the time dependence of the potential is in general immaterial.

Thus, under the condition

$$\omega_m \ll \min\left\{\nu_i,\ \frac{m\theta v_i}{r},\ \frac{m\theta^2 v_i^2}{r^2 \nu_i}\right\} \tag{5}$$

we can assume in first approximation that the particle moves in a static potential field with a potential

$$\Phi^{eff} = \sum_m \Phi_m(r) e^{im\phi}, \tag{6}$$

where the sum extends over all the harmonics m for which the condition (5) is satisfied.

The drift-velocity component normal to the magnetic surface, which is connected with the potential (6), is obviously equal to

$$v_n^i \approx \frac{c}{B} \frac{1}{r} \frac{\partial \Phi^{eff}}{\partial \phi} \tag{7}$$

and greatly exceeds the toroidal-drift velocity (3) at sufficiently large oscillation amplitudes[3]), when $e\Phi^{eff}/T_j \gg r/R$. If we assume, finally, that the

[3]) The case when the electric field does not depend on the time and amplitude of the azimuthal harmonic is sufficiently small, so that $e_j \Phi \sim (r/R) T_j$, was considered quantitatively in [5].

intensity of the oscillations is sufficiently high, so that the ratio $e_j\Phi^{eff}/T_j$ becomes of the order of unity and consequently the number of "trapped" particles (or "captured" by the wave) becomes of the order of the number of "transiting" particles, while the longitudinal velocity of either type of particle is of the order of thermal, then the displacement Δ_j will be of the same order in the entire region of collision frequencies $\nu_j \gg \omega_m$. Recognizing that the characteristic frequency of the azimuthal motion in this case is obviously equal to $\Omega_j \simeq m\theta v_j/r$, substituting formulas (7), (6), and (2) in (1), and averaging over the azimuth ϕ, we ultimately obtain

$$D \approx \nu_e \frac{v_e^2}{\omega_e^2}\frac{1}{\theta^2}\sum_m \frac{e_e^2|\Phi_m|^2}{2T_e^2}, \qquad (8)$$

$$\omega_m \ll \min\left\{\nu_e,\ \frac{m\theta v_e}{r},\ \frac{m\theta^2 v_e^2}{r^2\nu_e}\right\},$$

$$\kappa_j \approx \nu_{jj}\frac{v_j^2}{\omega_j^2}\frac{N}{\theta^2}\sum_m \frac{e_j^2|\Phi_m|^2}{2T_j^2}; \quad j = e, i,$$

$$\omega_m \ll \min\left\{\nu_{jj},\ \frac{m\theta v_j}{r},\ \frac{m\theta^2 v_j^2}{r^2\nu_{jj}}\right\}. \qquad (9)$$

From this it follows that for sufficiently intense oscillations, when $\sum_m(e_j^2|\Phi_m|^2/2T_j^2) \sim 1$, the toroidality plays no role, and the coefficients of diffusion and thermal conductivity increase appreciably (by a factor θ^{-2}) and are determined by the old classical formulas, in which the total field B should be replaced by the toroidal magnetic field B_ϕ.

On the other hand, if for any species of particles j one of the conditions (8) or (9) is violated, then the corresponding expression must obviously be replaced by the one previously obtained (see, e.g., [5]).

In conclusion, we note that the presence of harmonics of the potential of the longitudinal coordinate ζ violates the axial symmetry and can therefore lead under certain conditions to an even greater increase of the transport coefficient (as is the case in systems of "stellarator" type [3, 5, 6][4])).

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[4])We indicate that in Sec. 3 of [3] the frequencies of the Coulomb collisions ν_{jk} (j, k = e, i) must be taken to mean their effective values $\nu_{jk}^* = \nu_{jk}/2\varepsilon n I_n$, where the frequencies ν_{jk} are determined by formulas (8).

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POSSIBILITY OF PRODUCING HIGH PRESSURE IN A SOLID BY MEANS OF A STRONG-CURRENT ELECTRON BEAM

O.V. Bogdankevich and A.A. Rukhadze
P.N. Lebedev Physics Institute, USSR Academy of Sciences
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The problem of obtaining new materials in phase transitions frequently encounters the need for producing a static or dynamic pressure of the order of $10^5 - 10^6$ bar. To produce such high dynamic pressures in a solid one can use a strong-current relativistic electron beam. By focusing such a beam in a small volume at a certain depth from the surface of the solid, and by choosing the beam parameters and the target material in such a way that multiple ionization gives rise to a large number of free electrons, it is possible to obtain a state wherein the electron density is lower by one order of magnitude than in metals, i.e., $N_e \sim (1 - 3) \times 10^{23}$ cm^{-3}. In this case the Fermi energy will be on the order of $\varepsilon_F \simeq 0.5 \times 10^{-26}$ and $N_e^{2/3} \simeq 10 - 30$ eV. If we choose as the target a substance with readily-ionized atoms and closely-lying ionization levels (e.g., rare-earth elements or actinides, in which the first ionization potential is $\sim$5 eV and the next five - six levels are spaced 5 - 10 eV apart), then the temperature of the electron gas may turn out to be lower than the Fermi energy, and the gas is degenerate. To this end, obviously, it is necessary to satisfy the condition $T_e \lessgtr (\bar{I} - \varepsilon_F) < \varepsilon_F$, where I is the average ionization potential of the substance per electron. The pressure in such a degenerate electron gas is of the order of $p \simeq 2 \times 10^{-27} N_e^{5/3} \simeq (0.5 - 3)$ Mbar.

Let us estimate the beam parameters necessary to attain a pressure of 1 Mbar. Such a pressure is ensured by an electron concentration $N_e \simeq 1.5 \times 10^{23}$ cm^{-3}, corresponding approximately to fivefold ionization of the target atoms. The average ionization potential per electron in easily-ionized substances is in this case of the order of 25 - 30 eV, and the Fermi energy is $\varepsilon_F \simeq 15$ eV, i.e., $T_e \leq 10 - 15$ eV, and consequently the electron gas will be degenerate.

For complete fivefold ionization of a volume with linear dimension a the required energy is $\sim 4 \times 10^5 a^3$ J. Obviously, the dimension a should be, on the one hand, of the order of the beam electron mean free path, which amounts to several millimeters at an electron energy $\sim$3 - 5 MeV, and on the other hand, $a \ll V_F \tau_0$, where τ_0 is the duration of the beam pulse. The latter inequality ensures the absence of accumulation of space-charge-producing excess electrons in the volume. We shall use in the estimates $\varepsilon \sim 5$ MeV and a linear dimension $a \simeq 0.5$ cm. The necessary total beam energy is then 50 kJ. Each beam electron produces in this case 1.5×10^5 ionization electrons and, if it is recognized that the total number of beam electrons per pulse is $n \sim 10^{17}$, then the ionization results in 1.5×10^{22} electrons in a volume a^3, corresponding to the concentration $N_e \simeq 1.5 \times 10^{23}$ cm^{-3} needed to produce a pressure $p \simeq 1$ Mbar.

The pulse time τ_0 is, in turn, bounded from above. It can be easily understood that τ_0 must satisfy the condition $\tau_0 < a/v_0$, where v_0 is the velocity of the ions dragged by the electron-gas pressure, $v_0 \simeq 3 \times 10^5$ cm/sec. For the target and beam parameters assumed above, we have 3×10^{-9} sec $\ll \tau_0 <$ $< 2 \times 10^{-6}$ sec, then we obtain for the beam power $W \simeq 5 \times 10^{10}$ W, and for the