HIGH-FREQUENCY DETECTION OF GRAVITATIONAL WAVES

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The heretofore proposed gravitational-wave detectors are based on the interaction of the wave with a mechanical system [1 - 3] and therefore the frequency of the received gravitational wave cannot be too high. This frequency can be appreciably increased by using in lieu of masses electromagnetic fields of special configuration. We describe here a detector that realizes this possibility and makes it possible to receive waves of length $\sim 10^3$ - 1 cm. This detector employs a specific gravitational-electromagnetic resonance (GER), which makes possible a sensitivity $\sim 10^{-2}$ erg/sec-cm^2 or better.

The detector is an annular waveguide in which there propagates a train of electromagnetic waves filling the entire ring or a greater part of it. At two points of the ring, spaced 90° apart, there is continuous extraction of the electromagnetic waves, and the phase difference between the so-obtained signals is measured. If a plane polarized gravitational wave of frequency ω_g double the frequency of the revolution of the electromagnetic-wave train is incident perpendicular to the plane of the waveguide, then the phase difference should pulsate with an amplitude that increases in proportion to the square of the time.

To demonstrate this, let us consider neighboring sections of the electromagnetic-wave train in the waveguide. For simplicity, we can speak of two photons located at space-time points with separation $\delta\ell$, where the temporal component is $\delta\ell^0 = 0$. Let the photon pulses differ by δk (the comparison is with the aid of parallel transfer along the interval $\delta\ell$). It can be shown that after a time interval $\Delta x^0 = c\cdot\Delta t$ the difference between pulses changes by

$$\Delta\delta k^i = \Delta_0 \delta k^i - \frac{1}{k^0} R^i_{\cdot jk\ell} k^j \delta\ell^k k^\ell \Delta x^0 ,$$

where $\Delta_0\delta k$ dentoes the pulse-difference change due to the interaction with the waveguide, while the second term describes the action of the gravitational field.

In the rest system of the waveguide in the absence of a gravitational field, the photon energy is conserved. Consequently $\Delta_0\delta k^0 = 0$ and we obtain for δk^0 the equation

$$k^0 \frac{d\delta k^0}{dx^0} = - R^0_{\cdot jk\ell} k^j \delta\ell^k k^\ell . \qquad (1)$$

We put $\delta\ell^\alpha = n^\alpha\delta\ell$, where $\{n^\alpha\}$ is a unit vector (spatial) and $\delta\ell$ is a number. Then $k^\alpha = n^\alpha k^0$. Using this notation as well as the symmetry properties of the curvature tensor and the assumption that the gravitational field is weak, we obtain for δk^0 or, equivalently, for the frequency difference $\delta\omega_e$, the equation

$$\frac{d\delta\omega_e}{dt} = c\,\omega_e K_{\alpha\beta} n^\alpha n^\beta \delta\ell , \qquad (2)$$

where $K_{\alpha\beta} = R_{0\alpha 0\beta}$ is a symmetrical matrix.

We now consider the fact that the photon motion occurs under conditions of the annular waveguide. Consequently the vector n should rotate in a plane (say in the plane $x^3 = 0$) with a frequency $\omega_0 = c/r$, where r is the radius of the waveguide.

We introduce in place of $\delta \ell$ the angle distance $\delta\alpha$ between the photons, such that $\delta \ell = r\delta\alpha$, and assume that the principal axes of the matrix K coincide with the coordinate axes. We choose its eigenvalues in the form $\lambda_i = \Lambda_i \cos \omega_g t$. Rewriting Eq. (2) in terms of this notation and integrating it with respect to t, we can show that $\delta\omega_g$ contains a term proportional to the time

$$\delta \omega_e = -\frac{1}{4} t(\Lambda_1 - \Lambda_2) c\, \omega_e r \delta \alpha \cos 2\alpha \,. \tag{3}$$

This formula for the frequency shift is valid in the case when the compared sections of the electromagnetic wave lie close to one another. The formula for the final angle distance $\alpha_2 - \alpha_1$ can be obtained by integrating with respect to $\delta\alpha$:

$$\Delta\omega_e = -\frac{t}{2}\frac{c^2\omega_e}{\omega_g}(\Lambda_1 - \Lambda_2)\cos(\alpha_2 + \alpha_1)\sin(\alpha_2 - \alpha_1), \tag{4}$$

(we express r in terms of ω_g).

The frequency difference will give rise to accumulation of the phase shift of the electromagnetic oscillations in the two section. This shift is determined by integration with respect to the time:

$$\phi_2 - \phi_1 = \frac{t^2}{4}\frac{c^2\omega_e}{\omega_g}(\Lambda_1 - \Lambda_2)\cos(\alpha_2 + \alpha_1)\sin(\alpha_2 - \alpha_1)\,. \tag{5}$$

To obtain an estimate of the useful effect, it remains for us to express the quantity $\Lambda_1 - \Lambda_2$ in terms of the energy flux of the gravitational wave. Using the formulas of [4] for a plane gravitational wave propagating in the direction of the x^3 axis, we can express $\Lambda_1 - \Lambda_2$ in terms of the flux I of the gravitational energy:

$$\Lambda_1 - \Lambda_2 = \frac{4\omega_g}{c^2}\sqrt{\frac{2\pi G I}{c^3}}\,. \tag{6}$$

Substituting this expression in (8), we obtain ultimately

$$\phi_2 - \phi_1 = -\omega_e t^2\sqrt{\frac{2\pi G I}{c^3}}\cos(\alpha_2 + \alpha_1)\sin(\alpha_2 - \alpha_1). \tag{7}$$

This formula pertains to two points of the train of electromagnetic waves rotating with definite phases (α_1, α_2) relative to the gravitational wave. On the other hand, if one fixes the positions (β_1, β_2) of these points relative to the waveguide, then

$$\phi_2 - \phi_1 = -\omega_e t^2\sqrt{\frac{2\pi G I}{c^3}}\cos(\omega_g t - \beta_1 - \beta_2)\sin(\beta_2 - \beta_1), \tag{8}$$

i.e., the phase difference will vary sinusoidally with an amplitude that increases in proportion to the square of the time. The maximum amplitude (corresponding to $\beta_2 - \beta_1 = \pi/2$, i.e., to locating the pickoff points a quarter-circle apart) is equal to:

$$\Delta\phi_{max} = \sqrt{\frac{2\pi GI}{c^3}}\,\omega_e t^2 . \qquad (9)$$

Substituting in (9) $\omega_e = 6 \times 10^{10}$ sec^{-1} and t = 1 sec (this is the "ringing" time in superconducting microwave resonators [5]) we find that to measure a gravitational-radiation flux $I = 1 \times 10^{-2}$ erg/sec-cm^2 it is necessary to register $\Delta\phi = 1 \times 10^{-9}$ rad. It is clear that an increase in the Q of the superconducting resonators (or, equivalently, an increase of t) and also a decrease of the distinguishable phase shifts will lead to an increase of the sensitivity of such a detector. If ω_g is not equal to 2c/r, then the factor $\omega_e t^2$ in (9) should be replaced by ω_e/Ω^2, where $\Omega = \omega_g - (2c/r)$ is the beat frequency. We emphasize once more that such a relatively high sensitivity is the consequence of the resonance between the frequency of the gravitational wave and the frequency of rotation of the field in the resonator (GER).

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INVESTIGATION OF THE CHARACTER OF THE EMISSION OF A NEODYMIUM GLASS LASER WITH A PASSIVE SHUTTER HAVING A FINITE RELAXATION TIME

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The relation between the relaxation time τ_{rel} of the population of the levels of dyes used as passive shutters in lasers, and the duration Δt of the ultrashort pulses (USP) is of great importance for obtaining complete mode locking, i.e., separation on the axial radiation period of a single USP with duration $\Delta t \sim (c\Delta\nu)^{-1}$, determined by the width $\Delta\nu$ of the radiation spectrum.

The existing theoretical papers on mode locking in lasers with passive shutters presuppose an infinitesimally short relaxation time of the dye, i.e., $\tau_{rel} \ll \Delta t$. There has been little theoretical study of the operation of a passive-shutter laser with a finite relaxation time. It is therefore of interest to investigate the emission characteristics of the lasers in order to establish a relation between τ_{rel} and Δt for the presently used passive shutters in neodymium-glass lasers.

The purpose of the present study was to estimate the relaxation time of the widely-used polymethine dyes[1)] and to investigate experimentally the emission characteristics of lasers with such shutters.

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1) Soviet-made dyes known as Nos. 3955 and 1000.