Note added in proof. Afanas'ev, Belenov, Markin, and Poluektov (ZhETF Pis. Red. 13, 462, 1970 [JETP Lett. 13, 331, 1971] consider the population of the upper vibrational levels and the dissociation following excitation of the first vibrational level; they do not, however, consider the effect of anharmonicity.

[1] A. A. Ovchinnikov, Zh. Eksp. Teor. Fiz. 57, 263 (1969) [Sov. Phys.-JETP 30, 147 (1970)].
[2] J. Jortner and S. A. Rice, J. Chem. Phys. 44, 3364 (1966).
[3] V. M. Agranovich, Fiz. Tverd. Tela 12, 562 (1970) [Sov. Phys.-Solid State 12, 430 (1970].
[4] O. R. Bates and G. Poots, Proc. Roy. Soc. A66, 784 (1953).
[5] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 12, 525 (1942).

ANOMALIES OF THERMAL EMF IN PHONON-PHONON DRAGGING

V. A. Kozlov and E. L. Nagaev

Submitted 13 April 1971
ZhETF Pis. Red. 13, No. 11, 639 - 643 (5 June 1971)

The purpose of this research is to study the thermal emf of semiconductors under conditions of hydrodynamic flow of phonons under the influence of a temperature gradient applied to them. Under the indicated conditions, besides the usual effect of electron dragging by the phonons, another effect becomes quite significant, namely the dragging of the longer-wave phonons by thermal phonons, an effect to which no attention has been paid in the past. This effect, as will be shown below, can cause anomalously large values of the thermal emf, and even exponentially large values in the case of highly perfect crystals.

The physical picture is as follows: The thermal phonons with frequencies $\sim kT$ are scattered mainly by one another, since the conduction-electron density in intrinsic semiconductors is exponentially small. It is assumed that the frequency of the N-processes is large compared with the frequency of the U-processes. The result is a thermal-phonon distribution that is in equilibrium in a coordinate system moving with velocity u along the temperature gradient [1].

The conduction electrons interact only with phonons whose wavelength is comparable with their proper wavelength [2]. At an effective mass of the electron m on the order of the true mass and at $T > 0.1°K$ the frequency of such phonons (they shall be referred to as "electronic") is low compared with kT [3]. It is precisely the non-equilibrium nature of such "electronic" phonons which leads to the atomalies predicted by L. E. Gurevich for the thermal emf at low temperatures.

In calculations of the dragging effect it is usually assumed that the interaction of the "electronic" phonons with the thermal ones leads to dissipation of the momentum of the former [2, 3]. In fact, however, it can lead only to establishment of a drift of the "electronic" phonons with the same velocity as that of the thermal-phonon flow. Thus the thermal phonons, not being in equilibrium, drag the "electronic" phonons. On the whole, the dragging of the electrons by phonons is a two-step process: the conduction electrons are dragged by the "electronic" phonons which in turns are dragged by the thermal phonons.

The kinetic equations for the thermal and "electronic" phonons are written in the form

$$s\nabla T\frac{\partial F^{0}(\omega)}{\partial T} = I^{N}\{F\} + I^{U}\{F\}, \tag{1}$$

$$s\nabla T\frac{\partial G^{0}(\omega)}{\partial T} = I^{N}\{G, F\} + I^{U}\{G\}. \tag{2}$$

Here F and G are the distribution functions for the thermal and "electronic" phonons, respectively, and the index "0" denotes their equilibrium values. The symbols I^N and I^U denote the parts of the collision integral corresponding to processes with and without conservation of the total quasimomentum, and s is the phonon velocity. In writing down Eq. (1) for the thermal phonons it is recognized that the dominant role is played by their collisions with one another, and that the influence exerted on them by the "electronic" phonons can be neglected.

It is assumed that for thermal phonons the frequency ν_N^T of the N-collisions is large compared with the frequency ν_U^T of the U-collisions. If the scattering by defects can be neglected, then at low temperatures this condition is always satisfied.

As to Eq. (2) for the "electronic" phonons, since their number is small it suffices to take into account only their N-collisions with the thermal phonons. If the condition $\nu_N^T \gg \nu_U^T$ is satisfied for the thermal phonons, then the corresponding inequality $\nu_N^e \gg \nu_U^e$ should be satisfied for the electronic phonons. Indeed, if the quasimomentum dissipation is due to Umklapp processes, then both ν_N and ν_U depend on the phonon frequency ω in the same manner - they are proportional to ω^2. On the other hand if the dissipation is due to scattering by point defects, then ν_U decreases wtih decreasing frequency even more rapidly, like ω^4.

As was already indicated, the solution of (1) in the principal approximation in ν_U^T/ν_N^T is given by [1]

$$F_{q\ell} = [\exp\{(\omega_{q\ell} - vq)/kT\} - 1]. \tag{3}$$

The drift velocity v is determined by the equation

$$\frac{v}{r_U} = b \sum_\ell \int q^4 [(vq) F_0'(\omega)]\, dq = -\gamma \nabla T,$$
$$\gamma = \frac{b}{T} \sum_\ell \int q s \ell \omega_{q\ell} F_0'(\omega)\, dq, \quad b^{-1} = \sum_\ell \int q^2 F_0'(\omega_{q\ell})\, dq. \tag{4}$$

In an isotropic crystal the electrons interact only with longitudinal long-wave phonons. Bearing only such phonons in mind, we can omit the polarization index in (2). We seek the solution of this equation in the form

$$G_q = G_q^0 - \phi_q \frac{\partial G_q^0}{\partial \omega}. \tag{5}$$

In the interpretation of Eq. (2) it must be recognized that three-phonon processes, in which two thermal phonons are absorbed or emitted simultaneously, are impossible. Taking (3) and (5) into account, Eq. (2) takes then the form

$$s\omega_q \frac{\nabla T}{T} - \frac{(vq)}{r_q^N} = -\frac{\phi_q}{r_q^N},$$
$$\frac{1}{r_q^N} = \frac{2\pi}{\hbar} \sum_{q'\ell'\ell''} |\Phi(q; q+q')|^2 (F_q^3{}_{\ell'} - F_{q+q'\ell''}^0) \times \tag{6}$$
$$\times \delta(\omega_q + \omega_{q'\ell'} - \omega_{q''\ell''}),$$

where Φ is the matrix element for the corresponding three-phonon process. In (6), in accord with the condition $\nu_N^e \gg \nu_U^e$, the terms proportional to ν_U^e were discarded and the condition $\omega \ll kT$ was used.

The standard procedure for solving the kinetic equation for the electrons with allowance for the non-equilibrium character of the phonons [3, 4] leads to the following expression for the thermal emf in terms of the electron relaxation time for scattering by phonons, $\langle\tau\rangle$,

$$\alpha = \alpha_0 + \alpha_1 + \alpha_2,$$

$$\alpha_1 = \left(\frac{k}{e}\right)\frac{ms^2}{kT}\frac{\langle \tau_{ph}(p)\rangle}{\langle \tau\rangle}, \qquad \alpha_2 = \left(\frac{k}{e}\right)\frac{\gamma m}{k}\frac{\tau^U}{\langle \tau\rangle}, \tag{7}$$

$$\tau_{ph}(p) = \frac{1}{4p^4}\int_0^{2p} \tau_{ph}(q)\, q^3 dq, \qquad \langle F\rangle = -\frac{\int p^2 \frac{\partial f_p^0}{\partial \epsilon} F\, dp}{\int p^2 \frac{\partial f_p^0}{\partial \epsilon} dp}.$$

Here α_0 is the thermal emf obtained without allowance for the dragging of the electrons by the phonons, α_1 is the contribution to the thermal emf by the usual dragging of the electrons by the "electronic" phonons, under the assumption that the thermal phonons are in equilibrium, and α_2 is the contribution made to the thermal emf by the two-step dragging of the electrons by the electronic phonons and the dragging of the latter by the thermal phonons. Expressions for α_0 and α_1 were obtained earlier (see, e.g., [2, 3]). What is new is the term α_2.

To estimate the order of magnitude of α_2/α_1 it should be noted that for a Debye spectrum we have, according to (4), neglecting the velocity difference between the longitudinal and transverse phonons,

$$\frac{\alpha_2}{\alpha_1} = \frac{\tau^U}{\langle \tau_{ph}(p)\rangle}.$$

From the condition for hydrodynamic flow of the phonons $\nu_U^T \equiv \tau_U^{-1} \ll \nu_N^T$ it still does not follow that this ratio is large compared with unity, since $\langle \tau_{ph}(p)\rangle^{-1} = \nu_N^e \ll \nu_N^T$ is not necessarily large compared with τ^U. In principle, however, this inequality can be satisfied in very pure crystals. Then the effect of two-step dragging makes a contribution to the thermal emf much larger than the effect of ordinary dragging of electrons by phonons. In this case, in principle, it is possible to obtain exponentially large thermal emf $\sim\exp(\Theta_0/T)$, where Θ_0 is a characteristic temperature on the order of the Debye temperature.

Even if the condition $\nu_U^T \ll \nu_N^e$ is not satisfied, the relation $\nu_U^T \sim \nu_N^e$ is perfectly realistic. Even in this case the effect in question has a strong influence on the value of the thermal emf.

The authors are grateful to N. S. Lidorenko and I. B. Rubashov for stimulating the work. They are also grateful to R. N. Gurzhi and V. M. Kontorovich for a useful discussion of the problem.

[1] R. N. Gurzhi, Usp. Fiz. Nauk 94, 689 (1968) [Sov. Phys.-Usp. 11, 255 (1968)].
[2] C. Herring, Phys. Rev. 96, 1163 (1954).
[3] A. I. Ansel'm, Vvedenie v teoriyu poluprovodnikov (Introduction to the Theory of Semiconductors), Fizmatgiz, 1962.
[4] J. M. Ziman, Electrons and Phonons, Oxford, 1960.

GROUPS OF PARALLEL PENETRATING PARTICLES AND THEIR POSSIBLE SOURCES

M. F. Bibilashvili
Physics Institute, Georgian Academy of Sciences
Submitted 20 April 1971
ZhETF Pis. Red. 13, No. 11, 643 - 646 (5 June 1971)

In the investigations of groups of penetrating particles (GPP) at depths on the order of several hundred kg/cm^2, most of the particles forming these groups represent muons