$$a = a_0 + a_1 + a_2 ,$$

$$a_1 = \left(\frac{k}{e}\right)\frac{ms^2}{kT}\frac{\langle \tau_{ph}(p)\rangle}{\langle \tau \rangle}, \qquad a_2 = \left(\frac{k}{e}\right)\frac{\gamma m}{k}\frac{\tau^U}{\langle \tau \rangle}, \tag{7}$$

$$\tau_{ph}(p) = \frac{1}{4p^4}\int_0^{2p} \tau_{ph}(q) q^3 dq , \qquad \langle F \rangle = - \frac{\int p^2 \frac{\partial f_p^\circ}{\partial \epsilon} F dp}{\int p^2 \frac{\partial f_p^\circ}{\partial \epsilon} dp} .$$

Here α_0 is the thermal emf obtained without allowance for the dragging of the electrons by the phonons, α_1 is the contribution to the thermal emf by the usual dragging of the electrons by the "electronic" phonons, under the assumption that the thermal phonons are in equilibrium, and α_2 is the contribution made to the thermal emf by the two-step dragging of the electrons by the electronic phonons and the dragging of the latter by the thermal phonons. Expressions for α_0 and α_1 were obtained earlier (see, e.g., [2, 3]). What is new is the term α_2.

To estimate the order of magnitude of α_2/α_1 it should be noted that for a Debye spectrum we have, according to (4), neglecting the velocity difference between the longitudinal and transverse phonons,

$$\frac{a_2}{a_1} = \frac{\tau^U}{\langle \tau_{ph}(p) \rangle} .$$

From the condition for hydrodynamic flow of the phonons $\nu_U^T \equiv \tau_U^{-1} \ll \nu_N^T$ it still does not follow that this ratio is large compared with unity, since $\langle\tau_{ph}(p)\rangle^{-1} = \nu_N^e \ll \nu_N^T$ is not necessarily large compared with τ^U. In principle, however, this inequality can be satisfied in very pure crystals. Then the effect of two-step dragging makes a contribution to the thermal emf much larger than the effect of ordinary dragging of electrons by phonons. In this case, in principle, it is possible to obtain exponentially large thermal emf $\sim\exp(\Theta_0/T)$, where Θ_0 is a characteristic temperature on the order of the Debye temperature.

Even if the condition $\nu_U^T \ll \nu_N^e$ is not satisfied, the relation $\nu_U^T \sim \nu_N^e$ is perfectly realistic. Even in this case the effect in question has a strong influence on the value of the thermal emf.

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[1] R. N. Gurzhi, Usp. Fiz. Nauk 94, 689 (1968) [Sov. Phys.-Usp. 11, 255 (1968)].
[2] C. Herring, Phys. Rev. 96, 1163 (1954).
[3] A. I. Ansel'm, Vvedenie v teoriyu poluprovodnikov (Introduction to the Theory of Semiconductors), Fizmatgiz, 1962.
[4] J. M. Ziman, Electrons and Phonons, Oxford, 1960.

GROUPS OF PARALLEL PENETRATING PARTICLES AND THEIR POSSIBLE SOURCES

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In the investigations of groups of penetrating particles (GPP) at depths on the order of several hundred kg/cm^2, most of the particles forming these groups represent muons produced "trivially" in decays of pions or kaons in the atmosphere. It is a complicated matter to reveal some special events against such a background.

We have realized a new approach to the investigation of the nature of GPP. We investigated the correlations between the spatial and energy characteristics in these groups[1)] using data on the angular distributions.

Such a complete description of the GPP has become possible as a result of the "spark-calorimeter" procedure developed by us.

The results of the calculation are listed in the table.

Number of particles in group	Correlation coefficient		Limiting deviation to satisfy "0" hypothesis	Prob. of finding ρ in indic. limits
	sign of ρ	ρ ± σ		
2	–	0.074 ± 0,054	1,370	0,829
3	+	0,015 ±0,089	0,170	0,135
4	–	0,237 ± 0,112	2,116	0,966

It is seen from the table that the obtained correlation coefficient for three-particle groups cannot be regarded as significant, since a change in its value by only a factor of 0.17 causes ρ to go outside the limits of the critical region. By the same token, the "zero hypothesis" is satisfied and it must be assumed that ρ = 0.

Hoever, were the three-particle groups to consist entirely of trivially-produced muons, then the most general considerations, with allowance for the distribution of the transverse momenta in the production act, should lead to a correlation coefficient close to -1.

Let us construct the correlation field reflecting the connection between the energy and spatial characteristics in groups of this quantity, for the analysis of the causes leading to ρ = 0.

Figure 1 shows clearly four regions formed by the axes X = 0 and Y = 0. We shall arbitrarily call them the regions of small bursts - small distances, large bursts - small distances, small bursts - large distances, and large bursts - large distances.

Muons of trivial origin should be located in the second and third regions. It is seen,

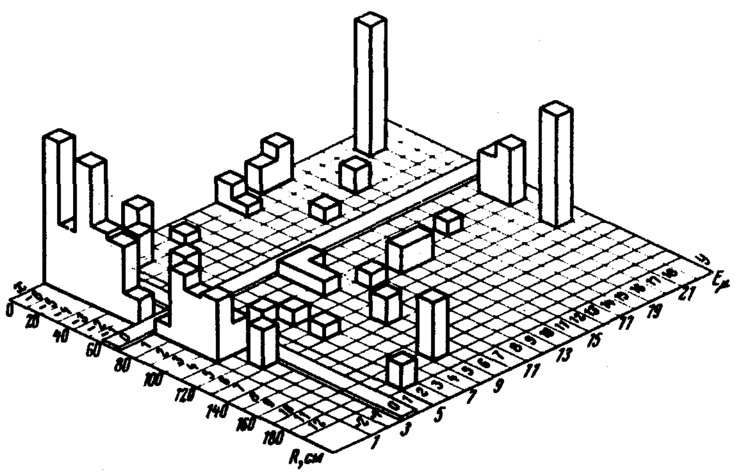

Fig. 1. Correlation dependence of the bursts and distances in three-particle groups.

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1) I.e., the statistical relation between the average burst produced by the group particles and the average distance between these particles.

however, that some of the cases lie also in the first and fourth regions.

The cases of the first region are formed principally by muon pairs produced by muons (tridents). A comparison of our experimental data with the results of the calculations in [2] with allowance for the threshold of our setup (0.5 GeV) shows them to be in good agreement.

Figure 2 shows the results of the calculations of [2] (family of curves), on which our data are superimposed. The upper point is the intensity of the cases of the first region in two-particle groups, and the lower point, in three-particle groups.

The family of continuous curves connects the intensity of the tridents with the depth of the detector (the parameter is the threshold energy of the detector itself) in three-particle groups. The dashed curves are for two-particle groups (pairs).

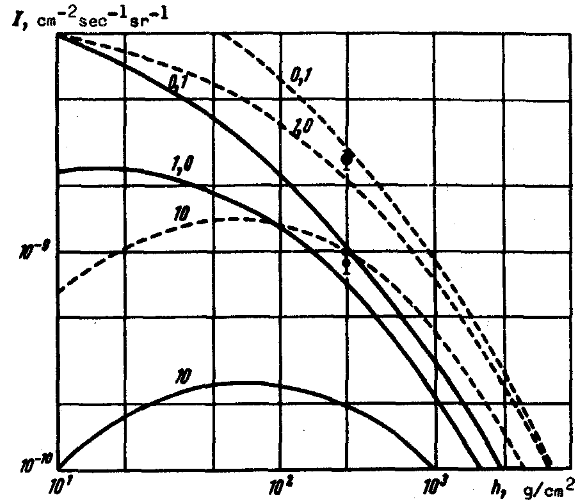

Fig. 2. Intensity of tridents in three-particle groups (lower point) and in pairs (upper point). Calculated intensity for triads (solid curves) and pairs (dashed). The parameter is the threshold energy of the muon-detecting devices.

The second and third groups, as already noted, are formed by muons of extensive air showers. This is confirmed also by registration of extensive air showers on the earth's surface simultaneously with detection of the cases of these regions under rocks. In addition, these regions are also characterized by a high degree of parallelism of the particles in the groups.

Let us turn now to the fourth region (large bursts - large distances). It is characterized by the fact that the particles in these groups have a perfectly measurable convergence in the ground above the apparatus.

Attention is called to the fact that when the magnitude of the burst changes by more than 1.5 orders of magntiude, the average distance between particles of this region remains practically unchanged.

Calculations have shown that this group of cases cannot be explained within the framework of the assume transverse-momentum distribution, with a mean value on the order of 0.4 GeV, unlike the second and third regions.

If the X-process considered in [3] is used to explain this nontrivial group of cases, then our experimental data make it possible to estimate the X-hadron mass. This mass, estimated from the mean distance between particles, turns out to be $\gtrsim 6$ GeV. As follows from [3], however, the value of the mass is closely related with the intensity of the phenomenon, and this makes it possible to estimate the mass by an independent method. This estimate, too, shows good agreement between [3] and our experiment.

Thus, the use of a spark calorimeter has made possible a new approach to the study of GPP, as a result of which it is shown that there are at least three processes responsible for their production. It is also shown that the completeness of the characteristics obtained therby makes it possible to estimate such an important parameter as the X-hadron mass.

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[1] M. F. Bibilashvili, Proc. 11th Internat. Conf. on Cosmic Rays, Budapest, 1969.
[2] S. R. Kel'ner and Yu. D. Kotov, ibid.
[3] J. D. Bjorken, S. Pakvasa, W. Simmons, and S. F. Tuan, Phys. Rev. 184, 1345 (1969).