NEUTRON STRENGTH FUNCTIONS

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Non-overlapping compound-nucleus resonances in the cross section for elastic scattering of slow neutrons are described by the strength function

$$SF = 2\pi\rho\Gamma_e\,, \tag{1}$$

where ρ is the average density of resonances with definite spin and parity, and Γ_e is the average elastic width. The quantity SR, as a function of the atomic weight, exhibits resonant forms connected with the existence of single-particle levels in the optical potential. The optical model with a complex potential describes qualitatively the position and the width of the form resonances. However, in the region of the minimum of the strength function ($A \sim 120$ for s-neutrons) the experimental values are markedly lower than those obtained using the optical model. The present article is devoted to a possible explanation of this discrepancy.

The form resonances can be regarded as resonances connected with excitation of the input (single-particle) states. If we neglect the interaction of the single-particle states with the more complex configurations, then the single-particle resonances are described by the Breit-Wigner formulas (provided the resonance widths $\Gamma_a(E)$ are small compared with the energy interval D between them). The values of the widths Γ_a and the positions of the resonances E_a are determined completely by the parameters of the real optical potential. In the same approximation, the states of complicated nature (compound-nucleus states) constitute superpositions of many-particle configurations, the simplest of which are configurations of the type two particles and on hole. The decay of these states to the continuous spectrum is due to the nuclear interaction H_{int}. In the region between the form resonances, where the connection between the single-particle and the compound-nucleus states can be neglected, the strength function coincides with its "background" value $(SF)_{bkg}$:

$$(SF)_{bkg} \equiv s = 2\pi\rho\gamma_e\,, \tag{2}$$

where γ_e is the average elastic width of the decay of the compound-nucleus states to the continuous spectrum as a result of the interaction H_{int}. Such a mechanism of the decay of the compound-nucleus level exists obviously also in the region of the single-particle resonance. In addition, in this region the compound-nucleus and the single-particle states become mixed because of the nuclear interaction H_{int}. The new compound-nucleus state produced by such a mixing can also decay into a continuous spectrum because of the admixture of the single-particle state. This provides one more mechanism for the decay of the compound-nucleus levels to the continuous spectrum. The decay amplitudes corresponding to the two indicated mechanisms are coherent [1, 2].

In the energy region where the elastic channel dominates, the levels of the compound nucleus do not overlap as a rule, and consequently the strength function can be obtained with the aid of the relation

$$SF = 1 - |\bar{S}|^2. \tag{3}$$

Here $\bar{S}$ is the scattering matrix averaged over an energy interval containing many levels of the compound nucleus.

The quantity $\bar{S}$ can be identified with diagonal elements of the S matrix obtained in the case of overlaping levels of the compound nucleus [1, 2]. Then we have the following expression for $\bar{S}(E)$ in the energy interval near an isolated single-particle resonance ($(E - E_a)^2$, Γ_a^2, $\Gamma_s^2 \ll D^2$):

$$\bar{S}(E) = e^{2i\delta_0(E)}\left[1 - \frac{1}{2}s(E) - \frac{i\left[\Gamma_a^{1/2} - \frac{i}{2}(s\Gamma_s)^{1/2}\right]^2}{E - E_a + \frac{i}{2}\Gamma_s}\right], \tag{4}$$

where $\Gamma_s >> \Gamma_a$ is the width of the "smearing" of the single-particle state $|a>$ over the levels of the compound nucleus and $\delta_0(E)$ is the nonresonant part of the potential-scattering phase. We note that no account is taken in (4) of the "external" mixing of the single-particle and compound-nucleus states [1, 2]. The "background" strength function s(E) is proportional to the width Γ_s:

$$s(E) = |A_0(E)|^2 B^{-2} \Gamma_s , \tag{5}$$

where $A_0(E)$ is the nonresonant part of the amplitude of the continuous-spectrum amplitude (normalized to a delta-function in energy); B^{-2} is a normalization factor equal to $2\pi R$ if $KR \equiv X >> 1$ (K is the wave vector of the nucleon inside the nucleus and $R = r_0 A^{1/3}$ is the radius of the nucleus).

An expression for $\overline{S}(E)$ in the energy interval $|E - E_a| \sim D$ can be obtained by summing pole terms of the type (4), resulting from each isolated single-particle resonance. In such a summation it must be borne in mind that the width of the decay of the single-particle state $|a(E_a)>$ to the continuous spectrum corresponding to the energy E is given by

$$\Gamma_a(E) = P(E)\, \gamma |< a(E) | a(E_a) > |^2 , \tag{6}$$

where P(E) is the penetrability of the barrier and is the reduced width and does not depend on the energy if $|E - E_a| \sim D << K^2/2m$.

Taking this remark into account, the summation over the resonances leads to the following expression for $\overline{S}(E)$:

$$\overline{S}(E) = e^{2i\delta_0(E)} \left\{ 1 - \frac{1}{2} s(E) - \frac{1}{2} P(E) \frac{i(1 - ib\xi)^2}{\mathrm{tg}(\pi/D)(E - E_a) + i\xi} \right\}, \tag{7}$$

where E_a is the energy of the single-particle level closest to E, $\xi = \pi\Gamma_s/2D$, and $S(E) = b^2 \xi P(E)$.

Using relations (3) and (7), we obtain the following expression for the strength function SF(E):

$$SF(E) = s(E) \frac{[\mathrm{tg}(\pi/D)(E - E_a) + b^{-1}]^2}{\mathrm{tg}^2(\pi/D)(E - E_a) + \xi^2} . \tag{8}$$

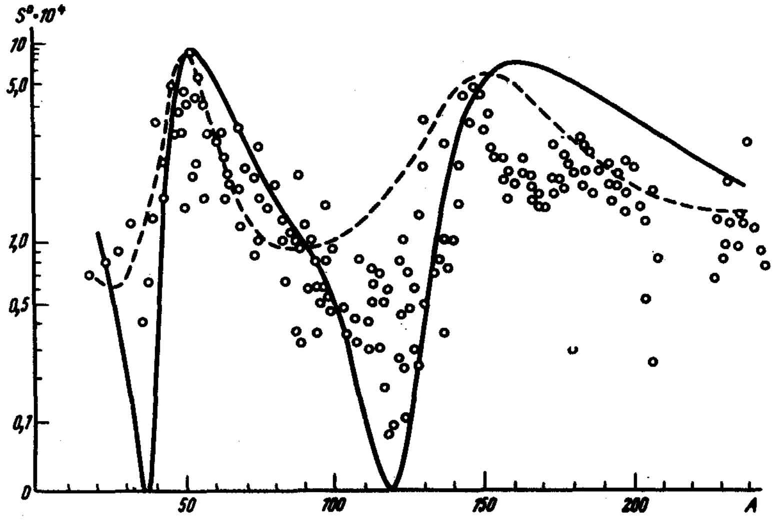
It follows from this expression that the strength function reveals an asymmetry, depending on the difference $E - E_a$, and vanishes at the value of $E_0 - E_a$ determined by the condition $b \tan (\pi/D)(E_0 - E_a) = -1$. At a fixed energy $E \to 0$, SF(R) vanishes for a nucleus with radius R_0 determined from the condition $b \tan (\pi/D) E_a(R_0) = 1$.

In the case of a square potential well relation (8) for $E \to 0$ can be written as

$$SF(R) = P\xi \frac{(\mathrm{ctg}\, \alpha A^{1/3} - 1)^2}{\mathrm{ctg}^2 \alpha A^{1/3} + \xi^2} , \tag{9}$$

where $\alpha = Kr_0$ and $P = 4kK^{-1}$ for s-neutrons. For a qualitative comparison with the experimental data and with the results of the optical model, it is possible to use formula (9) also for the case of a realistic potential. In this formula the parameter is $\alpha \sim 1$ and the penetrability P(E) is calculated for a potential with a diffuse edge, in accord with the definition given in [3].

Under the same assumptions, the expression for the strength function in accordance with the optical model with volume absorption $W = \Gamma_s/2$ is:



$$SF(R) = P\xi \frac{\operatorname{ctg}^2 \alpha A^{1/3} + 1}{\operatorname{ctg}^2 \alpha A^{1/3} + \xi^2}. \qquad (10)$$

Unlike (9), this expression has no essential asymmetry relative to its maxima, and does not vanish anywhere. The figure shows, together with the experimental data of [1], as functions of the atomic weight, the quantities $S_0(A) = (1/2\pi)SF(E = 1 \text{ eV})$ for s-neutrons, calculated from formulas (9) and (10) with the following parameters: $P = 1.5 \times 10^{-3}$, $\alpha = 2.8$, $\xi = \beta A^{1/3}$, $\beta = 0.09$. As follows from the figure, formula (9) explains the existence of a deep minimum in the strength function for slow s-neutrons at $A \sim 120$.

In conclusion we note that the unaccounted-for interaction of the single-particle resonances leads to a nonzero value of the strength function in the region of the minimum.

A more detailed derivation of the formulas in this article will be presented in a subsequent paper.

[1] D. F. Zaretskii and M. G. Urin, Yad. Fiz. 11, 361 (1970) [Sov. J. Nuc. Phys. 11, 202 (1970)].
[2] D. F. Zaretskii and M. G. Urin, ibid. 14, No. 3, 1971.
[3] J. Blatt and V. Weisskopf, Theoretical Nuclear Physics, Wiley, 1951.

POWER-LAW INCREASE OF AMPLITUDE OF SCATTERING OF LARGE-SPIN VIRTUAL PARTICLE WITH INCREASING ENERGY

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In the discussion of real processes one sometimes uses the amplitude off the mass shell (cf., e.g., [1]). In the present article we point out the dangers connected with such a reasoning. It has turned out that for scattering of large-spin particles the high-energy asymptotic amplitude off the mass shell differs greatly from the asymptotic amplitude on the mass shell. First, using the method of [2], we shall show that the real part of the amplitude for elastic scattering of a virtual scalar meson by a spinless target decreases with increasing energy no faster than s^{-2} (the notation is explained in the figure). The term "virtual" meson is meant in the sense of ordinary quantum field theory. With this, both the concept of the amplitude off the mass shell and the method of analytic continuation with respect to the particle mass are defined. For the validity of the reasoning that follows it is important that all particles have a nonzer rest mass. We write the dispersion representation with re-