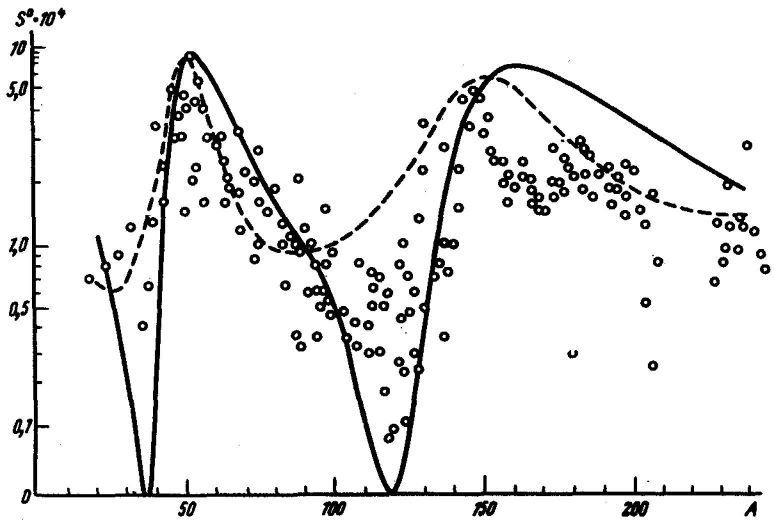

$$S F(R) = P \xi \frac{\mathrm{ctg}^2 \alpha A^{1/3} + 1}{\mathrm{ctg}^2 \alpha A^{1/3} + \xi^2}. \quad (10)$$

Unlike (9), this expression has no essential asymmetry relative to its maxima, and does not vanish anywhere. The figure shows, together with the experimental data of [1], as functions of the atomic weight, the quantities $S_0(A) = (1/2\pi)SF(E = 1\ \mathrm{eV})$ for s-neutrons, calculated from formulas (9) and (10) with the following parameters: $P = 1.5 \times 10^{-3}$, $\alpha = 2.8$, $\xi = \beta A^{1/3}$, $\beta = 0.09$. As follows from the figure, formula (9) explains the existence of a deep minimum in the strength function for slow s-neutrons at $A \sim 120$.

In conclusion we note that the unaccounted-for interaction of the single-particle resonances leads to a nonzero value of the strength function in the region of the minimum.

A more detailed derivation of the formulas in this article will be presented in a subsequent paper.

[1] D. F. Zaretskii and M. G. Urin, Yad. Fiz. 11, 361 (1970) [Sov. J. Nuc. Phys. 11, 202 (1970)].
[2] D. F. Zaretskii and M. G. Urin, ibid. 14, No. 3, 1971.
[3] J. Blatt and V. Weisskopf, Theoretical Nuclear Physics, Wiley, 1951.

POWER-LAW INCREASE OF AMPLITUDE OF SCATTERING OF LARGE-SPIN VIRTUAL PARTICLE WITH INCREASING ENERGY

L. L. Frankfurt
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

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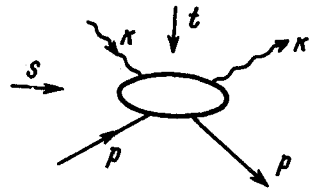
In the discussion of real processes one sometimes uses the amplitude off the mass shell (cf., e.g., [1]). In the present article we point out the dangers connected with such a reasoning. It has turned out that for scattering of large-spin particles the high-energy asymptotic amplitude off the mass shell differs greatly from the asymptotic amplitude on the mass shell. First, using the method of [2], we shall show that the real part of the amplitude for elastic scattering of a virtual scalar meson by a spinless target decreases with increasing energy no faster than s^{-2} (the notation is explained in the figure). The term "virtual" meson is meant in the sense of ordinary quantum field theory. With this, both the concept of the amplitude off the mass shell and the method of analytic continuation with respect to the particle mass are defined. For the validity of the reasoning that follows it is important that all particles have a nonzer rest mass. We write the dispersion representation with respect to the energy for the elastic scattering amplitude of the virtual meson:



$$A(s, t = 0, \; k^2) = \frac{1}{\pi}\int_{s_0}^{\infty} \frac{A_s(s', k^2)}{s' - s}\, ds' + \frac{1}{\pi}\int_{u_0}^{\infty} \frac{A_u(u', k^2)}{u' - u}\, du'. \quad (1)$$

Here $A_s(s, k^2)$ and $A_u(u, k^2)$ are the imaginary parts of A in the s- and u-channels. They are positive when k^2 is lower than the singularities. This follows from an examination of the Feynman diagrams and corresponds to a positive total cross section for the scattering of the virtual meson. Let us calculate the asymptotic real part under the condition that $A_s(s, k^2)$ and $A_u(u, k^2)$ decrease rapidly with increasing s and that

$$\int_{s_0}^{\infty} A_s(s', k^2)\, ds' = \int_{u_0}^{\infty} A_u(u', k^2)\, du'$$

$$\lim_{s\to\infty} s^2 A(s, k^2) = -\frac{1}{\pi}\Bigg\{\int_{u_0}^{\infty} A_u(u', k^2)(u' - u_0)\, du' +$$

$$+ \int_{s_0}^{\infty} A_s(s', k^2)[s' - s_0]\, ds' + \int_{u_0}^{\infty} A_u(u', k^2)[s_0 + u_0 - 2\mu^2 - 2k^2]\, du'\Bigg\}. \quad (2)$$

Here μ is the target mass. If we choose as the target a pion, the particle with the smallest hadron mass, then $s_0 + u_0 - 2\mu^2 \geq 0$. That is to say, $\lim_{s\to\infty} s^2 A(s, k^2) \neq 0$ when $k^2 \leq 0$. This result is a generalization of the statements made in [2, 3] to the case when the amplitude has poles below the two-particle threshold.

It can be shown analogously that the real par of the scattering amplitude of a virtual tensor meson increases with energy no slower than s^2. Let the tensor meson be elastically scattered by a spinless target (if the target has nonzero spin, then it is necessary to sum over its polarizations). Owing to PT invariance, the forward scattering amplitude is described by a tensor that is symmetrical in all four indices:

$$T_{\mu\lambda,\,\alpha\beta} = A_1 p_\mu p_\lambda p_\alpha p_\beta + A_2[\delta_{\mu\lambda}\delta_{\alpha\beta} + \dots] - A_3[\delta_{\mu\lambda} p_\alpha p_\beta + \dots]. \quad (3)$$

Here p is the target momentum and k is the momentum of the tensor meson. In formula (3) we did not write out explicitly the symmetrizing additional terms and the terms proportional to k. As the independent unit polarization vectors we can use when $k^2 < 0$

$$\epsilon^{\pm} = (1/\sqrt{2})(0, 0, 1, \pm i), \quad \epsilon^L = (k_z, k_0)/\sqrt{-k^2}. \quad (4)$$

These vectors have the useful orthogonality properties

$$(\epsilon^{\pm}, k) = (\epsilon^L, k) = 0, \; (\epsilon^e, \epsilon^n) = a_n \delta_{en},$$

where $a_n = -1$ for $\varepsilon^n = \varepsilon^{\pm}$ and $a_n = 1$ for $\varepsilon^n = \varepsilon^L$. We contract the amplitude (3) with the polarization vectors

$$\mathrm{Im}\, T^{++} = \mathrm{Im}\, T_{\mu\lambda,\,\alpha\beta}\epsilon^+_\mu \epsilon^+_\lambda(\epsilon^+_\alpha \epsilon^+_\beta)^* = 2\,\mathrm{Im}\, A_2,$$

$$\mathrm{Im}\, T^{+L} = \mathrm{Im}\, T_{\mu\lambda,\,\alpha\beta}\epsilon^+_\mu \epsilon^L_\lambda(\epsilon^+_\alpha \epsilon^L_\beta) = \mathrm{Im} A_3(p\epsilon^L)^2 - \mathrm{Im}\, A_2, \quad (5)$$

$$\mathrm{Im}\,T^{LL} = \mathrm{Im}\,T_{\mu\lambda,\,\alpha\beta}\,\epsilon^L_\mu \epsilon^L_\lambda (\epsilon^L_\alpha \epsilon^L_\beta)^* = \mathrm{Im}A_1(p\epsilon^L)^4 + 3\,\mathrm{Im}\,A_2 - 6\,\mathrm{Im}\,A_3(p\epsilon^L)^2.$$

All the imaginary parts in (5) are larger than zero, since this corresponds to positive cross sections for the scattering of polarized mesons. From these inequalities we readily get

$$\mathrm{Im}\,A_1(p\epsilon^L)^4 \geqslant 3\,\mathrm{Im}\,A_2 + 6[\mathrm{Im}A_3(p\epsilon^L)^2 - \mathrm{Im}A_2]. \qquad (6)$$

From the fact that Im A_1 is positive it follows (see formula (2)) that A_1 cannot decrease more rapidly than s^{-2}. We have thus shown that one of the amplitudes, T^{LL}, T^{+L}, or T^{++} increases when $k^2 \leq 0$ with energy no slower than s^2. Namely, in elastic scattering of a graviton, T^{++} increases with energy (T^{LL} and T^{+L} are equal to zero when $k^2 = 0$). The latter statement has been proved only in the lowest order in the gravitational interaction, since the reasoning is based on the absence of zero-rest-mass particles in the theory. It is possible to prove by the same method that when $k^2 \leq 0$ the real part of the scattering amplitude of any meson with spin J increases no slower than s^{2J-2}. For scattering of fermions with spin J, the direct use of such arguments leads to an asymptotic value of the type s^{2J-3}.

The power-law growth of the scattering amplitudes of virtual particles with increasing energy does not contradict the Froissart inequality [4], since for these particles there is no bilinear unitarity condition (continuation of the unitarity condition in the masses of the external particles does not affec the intermediate particles, which remain on the mass shell). This result can be strengthened by using the analyticity of A_1 in k^2. By $A_1(s, k^2)$ is meant here an invariant function in the scattering amplitude of the virtual meson with spin J, standing at the maximum number of target momenta. If A_1 decreases like s^{-2} when $k^2 \leq 0$, then the analyticity in k^2 and the limitations imposed by the unitarity condition (by the Froissart theorem) are compatible only in the coefficient of s^{2J-2} in the amplitude vanishes on the mass shell (it does not vanish identically in k^2 because of analyticity). If A_1 decreases more slowly than s^{-2} when $k^2 \leq 0$, then the amplitude near the mass shell may not contain a term that increases rapidly with the energy. An example is the function $s^{2J-2+\beta(k^2)}$, where $\beta(k^2)$ decreases with increasing k^2.

If the particles are reggeized, then the difficulties discussed above do not arise for in this case there is no mathematical object - amplitude off the mass shell, since the spin of the particle changes simultaneously with the mass. The results obtained in the article are a serious argument in favor of the point of view that the physical values for all processes should be expressed in terms of amplitudes of real processes.

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[1] R. Dashen and M. Gell-Mann, in: Proc. Coral Gables Conf. on Symmetry Principles at High Energy, 1966.
[2] B. Simon, Phys. Rev. D, 1, 1240 (1970).
[3] V. S. Vin and A. Martin, Phys. Rev. 135B, 1369 (1964).
[4] M. Froissart, Phys. Rev. 123, 1053 (1961).