[1] M.L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. 25, 296 (1953).
[2] H. Uberall, Phys. Rev. 103, 1055 (1956).
[3] G. Diambrini, Rev. Mod. Phys. 40, 611 (1968).
[4] A.I. Akhiezer and V.B. Berestetskii, Kvantovaya elektrodinamika (Quantum Electrodynamics), Moscow, 1969.

DISTRIBUTION OF ENERGY LOSSES FOR HEAVY POSITIVE PARTICLES IN A SINGLE CRYSTAL

N.P. Kalashnikov, V.S. Remizovich, and M.I. Ryazanov
Moscow Engineering Physics Institute
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The phenomenon of channeling of charged particles was discussed theoretically in a number of papers [1], but nonetheless the distribution function of the ionization losses of the energy under conditions of channeling has not yet been calculated. The purpose of the present article is to call attention to the fact that in the case of channeling of heavy particles there exists an approximation in which the inelastic processes can be described in an exceedingly simple manner.

Indeed, taking into account the smallness of the characteristic angles of the scattering of the heavy particle by an electron compared with the characteristic angles of elastic scattering of a heavy particle by a nucleus, it can be assumed that the scattering by an electron does not change the direction of motion of the heavy particle, and changes only its energy.

In this approximation, inelastic scattering does not change the distribution of the heavy particles over the cross section of the beam. Therefore, in the kinetic equation for the distribution function of the particles with respect to the energy loss Δ and the coordinates x and $\vec{r}_\perp$, the transverse coordinates $\vec{r}_\perp$ play the role of parameters:

$$\frac{\partial f(x, \mathbf{r}_\perp, \Delta)}{\partial x} = \int_0^\infty d\epsilon\, w_E(\epsilon, \mathbf{r}_\perp)\,[f(x, \mathbf{r}_\perp, \Delta - \epsilon) - f(x, \mathbf{r}_\perp, \Delta)], \tag{1}$$

In the case of small losses ($\Delta \ll E$) the probability of loss of energy ε by a particle per unit path $w_E(\varepsilon, \vec{r}_\perp) \simeq w_{E_0}(\varepsilon, \vec{r}_\perp)$, and the solution of (1) takes the form

$$f(x, \mathbf{r}_\perp, \Delta) = (2\pi i)^{-1} \int_{-i\infty+\sigma}^{+i\infty+\sigma} dp \exp\{-p\Delta - \int_0^\infty d\epsilon\, w_{E_0}(\epsilon, \mathbf{r}_\perp)[1 - e^{-p\epsilon}]\}. \tag{2}$$

The final particle-energy distribution function is obtained by averaging (2) over the spatial distribution of the transverse coordinates of the particles $W(\vec{r}_\perp)$ due to the elastic scattering

$$f(x, \Delta) = \int_s d^2\mathbf{r}_\perp\, W(\mathbf{r}_\perp)\, f(x, \mathbf{r}_\perp, \Delta), \tag{3}$$

where the integration over the transverse coordinates has been reduced to integration over the transverse cross section of one unit cell.

2. In an amorphous medium $W(\vec{r}_\perp)$ does not depend on $\vec{r}_\perp$ and (3) coincides with the particle-energy distribution function in an amorphous medium [2], $f_{am}(x, \Delta)$. In a single crystal, the flux of positively charged particles moves in such a way that the number of units between the crystallographic

planes is larger than near the planes ($\vec{r}_\perp = 0$), i.e., $W(\vec{r}_\perp) \geq W(0)$. Representing the spatial distribution of the particles in the single crystal in the form

$$W(r_\perp) = W_k(r_\perp) + W(0)$$

we can transform (3) into

$$f(x, \Delta) = f_k(x, \Delta) + \alpha f_{am}(x, \Delta),$$

where

$$f_k(x, \Delta) = \int d^2 r_\perp W_k(r_\perp) f(x, r_\perp, \Delta), \tag{4}$$

the quantity α is the fraction of the particles whose spatial distribution is homogeneous. For heavy particles, the characteristic values are $\Delta \gg \varepsilon$ in (2), and we can obtain

$$(1 - \exp(-p\epsilon)) \approx -p\epsilon + (1/2)p^2\epsilon^2. \tag{5}$$

It is important here that in integrating over the transverse coordinates in (4) we can neglect terms of order p^3 and higher, if the thickness L of the considered single crystal is bounded from above by the condition

$$n_0 L R_0^2 [e^4 (Z_1^{2/3} + Z_2^{2/3})/v^2]^2 \ll 1, \tag{6}$$

where R_0 is the distance from the crystallographic plane, on which $W(\vec{r}_\perp)$ differs noticeably from W(0). Substituting (5) in (4) and using the Lindhard distribution for $W_k(\vec{r}_\perp)$ [1], we obtain after integration

$$f_k(x, \Delta) = (\pi x \overline{\epsilon_k^2})^{-1/2} \exp\left\{ - \frac{(\Delta - \overline{\epsilon_k} x)^2}{\overline{\epsilon_k^2} x} \right\}, \tag{7}$$

where $\overline{\varepsilon_k}$ and $\overline{\varepsilon_k^2}$ are defined by the relations

$$\overline{\epsilon_k^2} = 4\pi n_0 Z_1 Z_2^2 e^4 (Z_1^{2/3} + Z_2^{2/3})/v^2, \tag{8}$$

$$\overline{\epsilon_k} = \frac{2\pi Z_1 Z_2^2 e^4}{E_{kin}} n_0 \left(\frac{M}{m_e}\right) \ln \frac{2 m_e v e^2 \sqrt{Z_1^{2/3} + Z_2^{2/3}}}{I_z} \tag{9}$$

where Z_1 is the atomic number of the crystal, I_Z is the ionization potential, and $Z_2 e$ and E_{kin} are the charge and kinetic energy of the incoming particle. The total distribution function consists therefore of two terms, one of which duplicates the distribution function in the amorphous body, multiplied by the factor $\alpha < 1$, and the second gives a Gaussian energy distribution with an average energy and width different from those of the amorphous body. Thus, from a comparison of $f_{am}(x, \Delta)$ and (7) it follows that the channeled particles lose on the average an energy smaller by a factor

$$\left(1 - \ln \frac{v}{e^2 \sqrt{Z_1^{2/3} + Z_2^{2/3}}} \Big/ \ln \frac{2 m_e v^2}{I_Z}\right) \tag{10}$$

than the non-channeled ones. In addition, for the channeled particles there decreases the dispersion of the energy losses by a factor

$$e^4 (Z_1^{2/3} + Z_2^{2/3}) / v^2. \qquad (11)$$

These results are connected with the absence of collisions with small impact parameters for the channeled particles.

It should be emphasized that expressions (7) - (9) were obtained under the assumption that the velocity of the incoming particles should be much larger than the velocity of the atomic electrons.

3. The obtained distribution function of the energy loss (4) and (7) is in satisfactory agreement with experiment [3]. In [3] there is given an experimental plot of the energy losses for incident protons with energy E_{kin} = 4.84 MeV, passing through a single crystal of Si with L = 2 mm, parallel to the [111] direction. This curve is characterized by two maxima, one of which coincides with the position of the maximum in complete absence of symmetry. This maximum is well described by the second term in (4).

The height of the second maximum in [3] exceeds by 9.1 times the height of the first maximum. The increase of the height of the second maximum is connected also with the decrease of the dispersion of the second maximum (11) and with the fact that the number of channeled particles is larger than that of the unchanneled ones. Therefore, using for α the experimental value 1.9, the distribution function of the energy losses (4) and (7) leads to an intensity ratio of the second maximum to the first equal to 10 (the experimental value is 9.1).

It was observed in experiment that the loss of the channeled particles is 0.45 of the loss of the unchanneled ones. In the considered approximation it follows from (10) that the decrease of the energy loss is 0.5. This is connected with the suppression of the "close" collisions, in which, according to Lindhard [1], there is lost approximately half of the total energy losses.

[1] J. Lindhard, Mat.-Fyz. Medd. Dan. Vid. Selsk 34, No. 14 (1965). Yu. Kagan and Yu. Kononetz, Zh. Eksp. Teor. Fiz. 58, 226 (1970) [Sov. Phys.-JETP 31, 124 (1970)].
[2] L.D. Landau, J. Phys. USSR 8, 201 (1944); L.D. Landau, Collected Works, Vol. 1, Nauka, 1969.
[3] C. Erginsoy, Brookhaven Lecture Series, No. 46, April 21, 1965.

TEMPERATURE STABILITY OF THE METALLIC MODIFICATION OF HYDROGEN

E.I. Estrin
Institute of Metallurgy and Metal Physics, Central Research Institute for Ferrous Metallurgy

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There have been lively discussions lately on the question of obtaining a metallic modification of hydrogen at high pressure. This modification, it is assumed, can have a high temperature T_c of transition to the superconducting state. For the practical utilization of the high T_c of the metallic hydrogen it is important to be able to maintain it at normal pressure at sufficiently high temperatures (on the order of or higher than T_c). In this connection, interest attaches to an estimate of the interval of stability of the metastable metallic hydrogen at normal pressure.