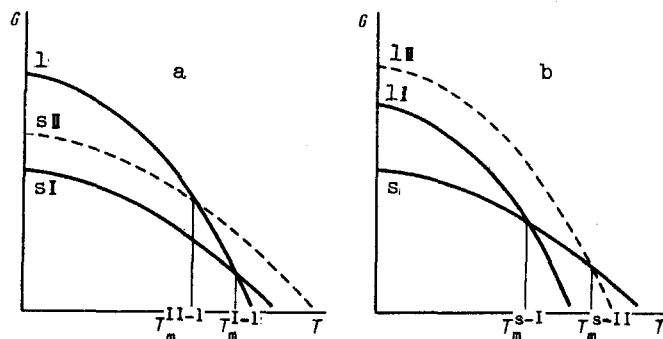Temperature dependence of the thermodynamic potential (G) of solid (s) and liquid (1) phases of the same substance: a - case of two solid phases (stable I and metastable II); b - case of two liquid phases (stable I and metastable II).

According to thermodynamics, the melting temperature of the metastable modification of any substance is always lower than the melting temperature of its stable modification (Fig. a). In the presence of two liquid phases (stable and metastable), the temperature of melting of the solid phase into a stable liquid phase is always lower than the temperature of melting into a metastable liquid (Fig. b). In the general case of two solid (stable and metastable) and two liquid (stable and metastable) phases, the lowest melting temperature is the temperature of melting of the metastable solid phase into a stable liquid phase.

In transformations in the solid state, one frequently observes during crystallization, by virtue of the kinetic factors, the metastable existence of one of the phases in the region of stability of the other phase. To the contrary, the experience accumulated to date shows that it is impossible to superheat a solid (at any rate by heating from the surface) above its melting temperature (possibly because the formation of the liquid layer on the surface of a melting body does not involve the loss of energy to the formation of the new surface).

As applied to hydrogen, the foregoing means, first, that the lowest melting temperature at normal pressure is the melting temperature of the metastable (metallic) modification into a stable (molecular) liquid (and this temperature consequently lies below the melting temperature of the stable (molecular) modification into a stable liquid) and second, that superheat of the metallic modification above the melting temperature is hardly possible.

Thus, the upper limiting temperature for the existence of the metallic modification of hydrogen at normal pressure obviously does not exceed the melting temperature of its usual modification ($14°K$). In fact, with decreasing pressure the phase transition from the metallic into the ordinary modification will apparently occur at a still lower temperature.

SURFACE SECOND SOUND IN LIQUID HELIUM

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Capillary waves can propagate over the surface of liquid helium, and in the superfluid solution He^3 in He^4 there exists also surface impurity levels [1, 2]. Both can be regarded as surface elementary excitations. Their motion is accompanied by transport of mass, energy, entropy, etc., and it can be regarded as a motion of a surface normal component.

The surface normal density is calculated in analogy with the volume case, and is equal to

$$\nu_n = -\frac{1}{2}\int p^2 \frac{\partial n_o}{\partial \epsilon} \frac{d^2 p}{(2\pi\hbar)^2}, \qquad (1)$$

where $\epsilon(\vec{p})$ is the energy of the surface excitations as a function of the two-dimensional momentum $\vec{p}$, and $n_0(\epsilon)$ is the equilibrium distribution function.

Substituting in (1) the spectrum of capillary waves $\epsilon = (\alpha p^3/\rho\hbar)^{1/2}$ (α, ρ – surface tension and density of the liquid) and the Planck function n_0, we obtain the surface normal density of pure He^4:

$$\nu_n = \frac{5\,T^{5/3}}{18\,\pi\hbar^2}\left(\frac{\rho\hbar}{\alpha}\right)^{4/3}\Gamma(5/3)\,\zeta(5/3), \qquad (2)$$

where $\Gamma(x)$ and $\zeta(x)$ are respectively the Gamma and the Riemann Zeta functions.

We note that the spectrum of the capillary waves does not satisfy the Landau superfluidity criterion. If, however, we take into consideration the finite nature of the force of gravity, then the critical velocity turns out to be finite.

In the case of a solution, the impurities situated on the surface levels are described by the dispersion law $\epsilon = -\epsilon_0 + p^2/2m_s$, where $\epsilon_0 \simeq 2°K$, $m_s \simeq 2m_3$ (m_3 is the mass of the He^3 atom), and the energy is reckoned from the minimum value of the impurity energy in the volume. The impurity part of the surface normal density is $\nu_n = m_s N_s$, where N_s is the number of impurities at the surface levels, and is calculated in [1].

As seen from (2), the surface normal density in pure He^4 is proportional to $T^{5/3}$. The volume at normal density at low temperatures, as is well known, is proportional to T^4. Therefore at sufficiently low temperatures, the influence of the volume normal component on the surface phenomena can be neglected and it can be assumed that there is only a surface normal density. This is particularly clearly manifest in weak solutions. The surface number of impurities increases exponentially with decreasing temperature, and the density of the impurities on the surface becomes atomic if the concentration c in the volume is of the order of $c \sim (T/\epsilon_0)^{1/2}\exp(-\epsilon_0/T)$. At $T = 0.1°K$, we get from this an uncontrollably small concentration of the order 5×10^{-10}. Thus, at temperatures on the order of one tenth of a degree there exists a dense layer of He^3 on the surface of practically pure He^4.

Let us consider the oscillations of a plane free surface with allowance for the surface normal component. The equation of motion of the surface can be written in the form

$$\frac{\partial i}{\partial x} = \rho'(v_{sz} - \dot{\zeta}), \qquad \frac{\partial i}{\partial t} - \frac{\partial a}{\partial x} = 0,$$

$$\dot{\sigma} + \sigma\frac{\partial v_{nx}}{\partial x} = 0, \qquad P = a\frac{\partial^2 \zeta}{\partial x^2}, \qquad (3)$$

$$\dot{N}_s + N_s\frac{\partial v_{nx}}{\partial x} = 0,$$

where the system of coordinates is chosen such that the x axis coincides with the direction of propagation of the oscillations, the z axis is normal to the unperturbed surface, $i = v_n(v_{nx} - v_{sx})$ is the surface momentum, α the surface tension, σ the surface entropy, P the pressure of the liquid, and the function $\zeta(x, t)$ determines the shape of the surface.

The first equation in (3) expresses the law of conservation of mass with allowance for the fact that the surface mass can always be made equal to zero by suitably defining the function $\zeta(x, t)$. The second equation is the law of conservation of the tangential momentum, since $\partial\alpha/\partial x$ is the tangential force. The third and fifth equations express the fact that the entropy and the impurities are transported only by normal motion. The fourth equation is the usual condition of equality of the normal forces.

The dependence of all the quantities on x and t is given by the factor $\exp(ikx - i\omega t)$. The potential of the superfluid velocity ϕ_s in the volume satisfies the Laplace equation, i.e., it is equal to $\phi_s = a \exp(ikx + kz - i\omega t)$, where a is the constant. In equation (3) it is therefore necessary to substitute $v_{sx} = ika$, $v_{sz} = ka$ and $P = -\rho\phi_s = i\omega\rho a$. Writing the deviation of the surface tension from the equilibrium value in the form

$$\delta\alpha = \frac{\partial\alpha}{\partial N_s}\delta N_s + \frac{\partial\alpha}{\partial\sigma}\delta\sigma$$

and eliminating δN_s, $\delta\sigma$, and a from (3), we obtain

$$i\,v_n v_{nx} + i\frac{\alpha k^2}{\omega}\left(1 - \frac{\omega^2\rho}{\alpha k^3}\right)\zeta = 0,$$

$$i v_{nx}\left\{\omega v_n + \frac{k^2}{\omega}\left(\frac{\partial\alpha}{\partial\sigma}\sigma + N_s\left(\frac{\partial\alpha}{\partial N_s}\right)\right)\right\} - i\frac{\alpha v_n}{\rho}k^3\zeta = 0, \qquad (4)$$

where we have neglected terms that are certainly small at small ω and k.

The system of equations (3) describes two types of oscillations. Oscillations of the first type occur at $v_{nx} \simeq 0$ and are ordinary capillary waves: $\omega_1^2 = (\alpha/\rho)k^3$. Oscillations of the second type occur at a practically immobile boundary $\zeta = 0$ and have an acoustic spectrum $\omega_2 = uk$ with velocity determined by the equation

$$u^2 = -\frac{\sigma}{v_n}\left(\frac{\partial\alpha}{\partial\sigma}\right)_{N_s/\sigma} = -\frac{N_s}{v_n}\left(\frac{\partial\alpha}{\partial N_s}\right)_{\sigma/N_s}. \qquad (5)$$

Oscillations of the second type are analogous to volume second sound, and we shall call them surface second sound.

Using the well-known [3] temperature dependence of the surface tension of He[4] and the equality $\sigma = -d\alpha/dT$, we find the velocity of the surface second sound in pure He[4]:

$$u^2 \simeq \frac{63}{40} \frac{\Gamma(7/3)\,\zeta(7/3)}{\Gamma(5/3)\,\zeta(5/3)} \left(\frac{\alpha T}{\hbar \rho}\right)^{2/3}$$

In the case of a solution, the surface tension has been calculated in [1] and substitution in (5) gives $u = (2T/m_s)^{1/2}$, which corresponds to a speed of sound in a two-dimensional monotonic ideal gas. This result is valid at high temperatures, when the impurities are far from degeneracy. If the impurities are strongly degenerate, then the velocity of the surface second sound is equal to $u = (N_s/m_s^*)(\partial\mu/\partial N_s)$, where μ is the chemical potential of the impurities, m_s^* is the effective mass, which differs from m_s because of the Fermi-liquid interaction between the impurities. Since at $T = 0$ the velocity u is of the order of the velocity of ordinary sound, it is clear that the function $u(T)$ has a minimum at a certain temperature.

The presence of a volume normal component leads to a certain damping of the surface sound. This damping is small if the frequency is not too low. Namely, the following two conditions should be satisfied

$$\omega \gg c\left(\frac{M^2}{\nu_n^2 a_o^5}\right)\sqrt{\frac{T}{M}}; \quad \omega \gg \left(\frac{\theta}{\hbar}\right)\left(\frac{M}{\nu_n a_o^2}\right)\left(\frac{T}{\theta}\right)^7 \ln\left\{\frac{T}{\hbar s}\left(\frac{\alpha}{\rho g}\right)^{1/2}\right\},$$

where a_0 is the interatomic distance, Θ is the Debye temperature of the liquid helium, s is the velocity of the second sound, M is the effective mass of the impurities in the volume, and g is the acceleration of free fall.

At $T \simeq 0.1°K$ and at concentrations $c \sim 10^{-6} - 10^{-8}$, the surface normal component can be regarded as atomic, i.e., $\nu_n \sim M/a_0^2$, and the written formulas lead respectively to the conditions $\omega \gg 10^3 - 10^5$ and $\omega \gg 10^{-1}$ (we have put $\Theta \simeq 10°K$).

An experimental study of the surface second sound would be of great interest, since it would make it possible to clarify the thermodynamic and kinetic properties of a two-dimensional Fermi liquid.

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ACCUMULATING NONLINEAR OPTICAL EFFECTS IN A PUMP FIELD WITH A BROAD FREQUENCY SPECTRUM

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1. In the present article we discuss the possibility of realizing an experimental growth of the intensity of light waves as a result of nonlinear interaction (Raman or parametric) with broad-band incoherent pumping.

2. Theoretical investigations of Raman and parametric processes in a noise pumping field carried out to date, do not give the complete picture of the