$$u^2 = \frac{63}{40}\frac{\Gamma(7/3)\,\zeta(7/3)}{\Gamma(5/3)\,\zeta(5/3)}\left(\frac{\alpha T}{\hbar\rho}\right)^{2/3}$$

In the case of a solution, the surface tension has been calculated in [1] and substitution in (5) gives $u = (2T/m_s)^{1/2}$, which corresponds to a speed of sound in a two-dimensional monotonic ideal gas. This result is valid at high temperatures, when the impurities are far from degeneracy. If the impurities are strongly degenerate, then the velocity of the surface second sound is equal to $u = (N_s/m_s^*)(\partial\mu/\partial N_s)$, where μ is the chemical potential of the impurities, m_s^* is the effective mass, which differs from m_s because of the Fermi-liquid interaction between the impurities. Since at T = 0 the velocity u is of the order of the velocity of ordinary sound, it is clear that the function u(T) has a minimum at a certain temperature.

The presence of a volume normal component leads to a certain damping of the surface sound. This damping is small if the frequency is not too low. Namely, the following two conditions should be satisfied

$$\omega >> c\left(\frac{M^2}{\nu_n^2 a_o^5}\right)\sqrt{\frac{T}{M}}; \quad \omega >>\left(\frac{\theta}{\hbar}\right)\left(\frac{M}{\nu_n a_o^2}\right)\left(\frac{T}{\theta}\right)^7 \ln\left\{\frac{T}{\hbar s}\left(\frac{\alpha}{\rho g}\right)^{1/2}\right\},$$

where a_0 is the interatomic distance, Θ is the Debye temperature of the liquid helium, s is the velocity of the second sound, M is the effective mass of the impurities in the volume, and g is the acceleration of free fall.

At $T \simeq 0.1°K$ and at concentrations $c \sim 10^{-6} - 10^{-8}$, the surface normal component can be regarded as atomic, i.e., $\nu_n \sim M/a_0^2$, and the written formulas lead respectively to the conditions $\omega >> 10^3 - 10^5$ and $\omega >> 10^{-1}$ (we have put $\Theta \simeq 10°K$).

An experimental study of the surface second sound would be of great interest, since it would make it possible to clarify the thermodynamic and kinetic properties of a two-dimensional Fermi liquid.

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ACCUMULATING NONLINEAR OPTICAL EFFECTS IN A PUMP FIELD WITH A BROAD FREQUENCY SPECTRUM

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1. In the present article we discuss the possibility of realizing an experimental growth of the intensity of light waves as a result of nonlinear interaction (Raman or parametric) with broad-band incoherent pumping.

2. Theoretical investigations of Raman and parametric processes in a noise pumping field carried out to date, do not give the complete picture of the

phenomenon. The theory of SRS and SMBS in an incoherent pumping field has been developed only for a nondispersive medium (see [5 - 7]). There are only partial results with respect to parametric amplification [8, 9].

We have developed a theory [13] of parametric and Raman processes in an incoherent pumping field, with account taken of the finite relaxation time of the nonlinearity and the dispersion of the medium. It is shown that although the broadening of the pump spectrum, generally speaking, lowers the efficiency of the indicated processes, an increase of the spectral density of the pumping is capable of compensating for this effect[1]), namely, the effective band $\Delta\omega_p^{eff}$ of the frequency (or angular) spectrum of the pump, involved in the process of amplifying a given spectral component of the signal, is determined by the ratio of the nonlinearity of the medium to its dispersion and is directly proportional to the average spectral density of the pump.

3. Nonstationary SRS in a given pump field $A_p[t - (z/u_p)]$ is described by equations for the complex amplitude of the Stokes wave A_s and the nondiagonal element of the density matrix σ [6]:

$$\frac{1}{u_s}\frac{\partial A_s}{\partial t} + \frac{\partial A_s}{\partial z} = \gamma_1 \sigma A_p\left(t - \frac{z}{u_p}\right), \tag{1a}$$

$$\frac{\partial \sigma}{\partial t} + \frac{1}{T_2}\sigma = \gamma_2 A_s A_p^*\left(t - \frac{z}{u_p}\right). \tag{1b}$$

The character of the scattering process depends on the relation between $\Delta\omega_p$ and the characteristic band $\Delta\omega = (\nu z)^{-1}$, which is determined by the length of the scattering region z and by the dispersion of the velocities $\nu = |u_s^{-1} - u_p^{-1}|$. When $\Delta\omega_p \ll \Delta\omega$ (regardless of the ratio between $\Delta\omega$ and $\Delta\omega_{sp} = 2/T_2$), the average intensity of the Stokes wave increases exponentially with increasing distance,

$$\langle |A_s^2(t, z)| \rangle = I_s(z) = I_{s0} \exp G_{coh} z, \quad G_{coh} = g_0 I_p, \tag{2}$$

where $g_0 = 2\gamma_1\gamma_2 T_2$ (all the parameters are henceforth normalized so that the quantities $\langle |A_{s,p}^2| \rangle = I_{s,p}$ have the dimension of the power density). When $\Delta\omega_p \gg \Delta\omega_{sp}$, $\Delta\omega$ the pump amplitude in (1) can be regarded as a δ-correlated random process:

$$\langle A_p(t) A_p^*(t') \rangle = 2\pi S \delta(t - t'), \quad S = I_p/\Delta\omega_p, \tag{3}$$

and the Stokes field can be regarded as a Markov process. The solution of Eqs. (1), obtained by reducing them to a Fokker-Planck equation, makes it possible to obtain the following expression for the intensity of the Stokes wave in the cross section z:

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1)The foregoing does not pertain to the case when the pump parameters vary in a regular fashion: for example, in the case of a sufficiently rapid linear variation of the pump there occurs a sharp decrease of the gain for any pump level [10].

$$I_s(z) = I_{av} \exp G_{incoh} z, \quad G_{incoh} = \frac{g_0}{\frac{2}{\pi} - S g_0 \nu^{-1}} \frac{\Delta\omega_{sp}}{\Delta\omega_p} I_p. \tag{4}$$

From a comparison of (2) and (4) it follows that $\Delta\omega_p^{eff} = \Delta\omega_{sp}/(2/\pi - Sg_0\nu^{-1})$ increases rapidly with increasing spectral density of the pump (see the figure).

At $S = S_{cr} = 2\nu/\pi g_0$, expression (4) has a singularity, and near the critical value of S the line width of the Stokes component is

$$\Delta\omega_s = \frac{4}{\pi} \Delta\omega_p^{eff} .$$

The validity of the Fokker-Planck approximation is limited to the region $\Delta\omega_s < \Delta\omega_p (S < S_{cr})$, i.e., $(\Delta\omega_p^{eff})_{max} \approx (\pi/4)\Delta\omega_p$. In the region $S > S_{cr}$ the quasistatic formula (2) is valid (see the figure). Analogous results can be obtained also for other types of scattering, particularly for SMBS and STS.

4. We consider now parametric amplification (PA) in a pump field randomly modulated in space and in time. If the average signal and pump frequencies differ by a factor of 2, we can write for the complex amplitude A_s of the amplified signal and the expression [8, 9]

$$\frac{\partial A_s}{\partial z} + \frac{1}{v_s} \frac{\partial A_c}{\partial t} + \beta \frac{\partial A_s}{\partial x} = \gamma A_p \left(t - \frac{z}{v_p}, x\right) A_s^* e^{i\Delta z}, \tag{5}$$

where $\Delta = k_p - 2k_s$ is the difference between the average wave vectors. In the essentially nonquasistatic case ($\Delta\omega_p = \tau_{coh}^{-1} \gg (\nu z)^{-1}$; the width of the angular spectrum of the pump is $\Delta\theta = \tau_{coh}^{-1} \gg \Delta\theta_{synch} = (\beta z)^{-1}$), the pump can be regarded as a δ-correlated process in space and in time. Then the average intensity of the signal wave in the section of the nonlinear medium (at the input, z = 0, the signal wave is assumed to be plane and monochromatic)

$$\langle | A_s^2 (t, x, z)| \rangle = I_s(z) = I_{s0} \exp\{2[\gamma^2 I_p \ell_{coh} - \delta]z\},$$

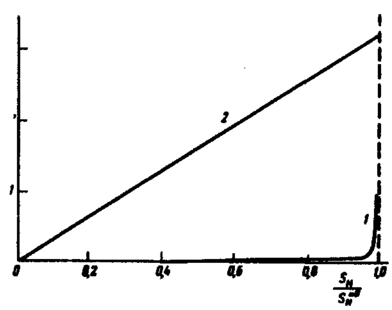
$$\ell_{coh} = \left(\frac{\nu}{\tau_{cor}} + \frac{\beta}{r_{cor}}\right)^{-1} \tag{6}$$

(in the field of monochromatic pumping at $\Delta = 0$

$$I_s(z) = I_{s0} \exp\{2(\gamma\sqrt{I_p} - \delta)z\}).$$

For the effective width of the frequency spectrum of the pump we obtain in this case $\Delta\omega_p^{eff} = (2\pi\gamma/\nu)^2 S$.

5. For the calculations, we write the formula for S_{cr} in the following form: for



Relative effective width of the pump spectrum $\eta = \Delta\omega_p^{eff}/\Delta\omega_p$, contributing to the stimulated Raman scattering (curve 1) and to parametric amplification (curve 2), as a function of the average spectral density of the pump.

forward SRS $S_{cr} = (I_p/\Delta\nu_p)_{cr} = 4\nu'/g_0$ (here $\Delta\nu_p$ is the pump band width in cm^{-1}, $\nu' = c\nu$). For backward SRS and SMBS, $S_{cr} = 8/g_0$. In the optical band, for forward SRS $\nu' = 10^{-2}$, $g_0 \approx 10^{-2}$ cm/MW and $S_{cr} = 4$ MW/cm. Thus, SRS forward in the field of a multimode laser with a very broad spectrum $\Delta\nu_p = 100$ cm^{-1} (such band widths are characteristic of multimode lasers using glass or dyes) is just as effective as in the field of single-mode pumping, at relative high power density $\approx$400 MW/cm^2. For SRS backward at the same data we have $S_{cr} \approx$ 800 MW/cm, leading to a strong forward-backward asymmetry. For SMBS in CS_2 we have $S_{cr} \approx 160$ MW/cm and consequently it is easy to obtain $\Delta\nu_p^{eff} \approx \Delta\nu_p \approx 100$ cm^{-1}.

6. For parametric amplification $S_{cr} = \Delta\nu_p(\nu'/\gamma)^2$. For crystals of the KDP type in the visible range $\gamma^{-2} \sim 25$ MW, $\nu' \sim 10^{-2}$, and $S_{cr} \approx (\Delta\nu_p/400)$ MW/cm. For $\Delta\nu_p \sim 100$ cm^{-1} we have $I_{p,\,cr} \approx 25$ MW/cm^2. Thus, the effective pumping of parametric light generators in the visible and infrared regions is possible with the aid of multi-mode lasers. The latter make it possible to broaden the class of sources of parametric light generator pumping and the more effective utilization of nonlinear crystals. In the ultraviolet band we can use for pumping parametric light generators laser based on xenon [1] and hydrogen [2]. In the $BeSO_4$ crystal, which is transparent up to 1700 Å [12], the use of a band width $\sim 5 \times 10^2$ cm^{-1} of the xenon laser is possible at a power density $\sim 10^3$ MW/cm^2. The threshold intensity of the pump, I_{thr}, for this crystal is smaller ($I_{thr} \approx 10 - 30$ MW/cm^2). The requirement imposed on the pump source can be determined from the plots of the diagram ($I_{thr} = S_p\Delta\nu_p^{eff}$). A multi-frequency hydrogen laser can be used to pump a parametric light generator using centrally-symmetrical crystals such as MgF_2 and LiF in an external static field. It is necessary, here, to have a power density $\sim 10^4$ MW/cm^2. In the x-ray band in the region where coherent three-photon decay is observed [11], we have $\nu' \approx 10^{-8}$; this apparently does not suffice for the compensation of the drop of the nonlinearity, and also for the use of megaampere [3] electron beams in pump sources.

7. Interest in the questions discussed here is connected mainly with problems of nonlinear optics in the ultraviolet and x-ray bands, where the degree of coherence of the powerful sources is not large (see [1 - 3]). Analogous problems arise in the optics of the visible and infrared bands, when the pump used in multi-mode radiation or superluminescence, and also in astrophysics, particularly in connection with the possibility of stimulated scattering of intense incoherent light in cosmic plasma (see [4]). Another fundamental problem is the extent to which parametric generators and amplifiers with broad-band pumping are capable of replacing lasers in those regions of the spectrum where the obtaining of laser action is difficult.

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ERRATA

In the article by V.A. Kuryavtsev and E.M. Levin (Vol. 13, No. 9), an error was made in the separation of the physical state on the third daughter trajectories. Therefore formulas (12) and (13) are incorrect. The correct expressions correspond to two states that enter in the amplitude with positive residues for all $j > 5$, and therefore there are no ghosts on the entire third trajectories.

All the remaining conclusions concerning the spectrum of the states on the second and third daughter trajectories remain in force.

In the article by E.M. Epshtein (Vol. 13, No. 9) there are misprints. Formula (2) on page 364 reads

$$\left| \frac{q}{2} - \frac{2m\Omega}{q} \right| < p_F ,$$

and should read

$$\left| \frac{q}{2} - \frac{m\Omega}{2} \right| < p_F .$$

On page 365, line 7, $|\alpha| \sim 10\ \mathrm{cm}^{-1}$ should be replaced by $|\alpha| \sim 10^3\ \mathrm{cm}^{-1}$.