

between them has not yet been fully treated in the theory.

An investigation of the transverse magnetoresistance of n-InAs in the same region of temperatures and fields disclosed no noticeable oscillations.

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HARD VAVILOV-CERENKOV RADIATION IN A SINGLE CRYSTAL

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1. The classical condition for the occurrence of Cerenkov radiation, $v = (c/n)\cos^{-1}\theta$ $\equiv u < c$ (c is the velocity of light in vacuum, v the electron velocity, θ the angle between its vector and the photon emission direction, and n the refractive index of the medium) [1], cannot be satisfied in the x-ray band, where $n \lesssim 1$, so that it is customary to assume that "radiation in this region is impossible" [2], page 29). This statement is valid only so long as it is permissible to describe the properties of the medium with the aid of a single macroscopic parameter - the refractive index n .

2. When considering the propagation of x-rays in a single crystal, the monochromatic wave field of radiation can be represented as a sum of an infinite number v of spatial harmonic components of a three-dimensional Fourier series of the same frequency v , but with different directions and propagation velocities of the phase u_{klm} [3]. Among the set of spatial harmonics, there exist some for which $u_{klm} < c$.

3. The quantity u which enters in the synchronism condition (Sec. 1) is essentially the rate of displacement of the phase of electromagnetic wave in the direction of electron motion. Therefore, if the velocity v of an electron moving in the single crystal coincides in magnitude and direction with the velocity $u_{klm} < c$ of one of the spatial harmonics ($v = u_{klm}$), then we can expect Cerenkov radiation to occur in the x-ray band.

4. It can be shown [4] that for electromagnetic oscillations of x-ray frequency propagating in a single crystal there exist resonant modes represented by the same point on the boundaries of the Brillouin zones [3], where the gradient of the frequency with respect to the

wave vector vanishes. The natural frequencies in the direction of propagation of the resonant modes can be found from the condition for crossing of the Ewald propagation sphere [3], with not less than two reciprocal-lattice points, disregarding the point at the origin. In the limiting case when one of the points and the origin are located on opposite ends of a sphere diameter, it is sufficient to cross only this single point. For photons with energy on the order of several dozen keV, the intrinsic Q of the resonant modes of the single crystal, determined by the photoeffect losses, Compton scattering, and scattering by phonons, can reach a value of the order of 10^8 [4].

5. By virtue of the noted resonant properties of the single-crystal lattice, the spectrum of the hard Cerenkov radiation produced in it consists of lines coinciding with those resonant modes, for which the synchronism condition $v = u_{k\ell m}$ is satisfied, and of a less intense nonresonant background. The increased intensity of the resonant lines is due essentially to stimulated emission.

6. By way of an illustrative example it is useful to consider a two-dimensional square lattice with constant d . From the Ewald construction follows a condition for the natural frequency $(2d\nu_{\text{ср}}/c)^2 = \alpha^2 + \beta^2$ (α and β are integers which do not vanish simultaneously), and a condition that specifies the direction of propagation of radiation relative to one of the crystal axes, $\cos \theta = \beta (\alpha^2 + \beta^2)^{-1/2}$. If the electron velocity v is directed along the same axis, then the velocity of displacement of the phase of the wave with frequency $\nu_{\text{ср}}$ along its trajectory is equal to $u_m^{\alpha\beta} = (\alpha^2 + \beta^2)^{1/2} (\beta + 2m)^{-1}$ ($m = 0, \pm 1, \pm 2, \dots$). The synchronism condition (Sec. 1) is satisfied in the form $v = c(\alpha^2 + \beta^2)^{1/2} (\beta + 2m)^{-1}$, where the values $\alpha = 0$ (the limiting case of zero intensity) and $m = 0$ (impossibility of synchronism) are excluded. Example: $\alpha = 1, \beta = 0, c/\nu_{\text{ср}} = 2d$ (of the order of 5 \AA), $m = 1, v/c = 0.5$ (electron energy approximately 80 keV). In the case of non-integer values of α and β , the foregoing formulas describe also the conditions for the occurrence of the nonresonant background.

7. The applicability of the foregoing analysis (but not at all the possibility of occurrence of the phenomenon itself) is limited by the fact that we have neglected both the relativistic and quantum corrections. The former set an upper limit and the latter a lower limit on the velocities $v = u_{k\ell m}$ (the quantum effects can be disregarded so long as the de Broglie wavelength of the electron remains much smaller than c/v and d ([2], page 35)). The electrons from the preceding example, with energies on the order of 80 keV, do not go beyond the two limits.

8. In conclusion it is useful to note that an analogous phenomenon, the radiation of electromagnetic oscillations when an electron beam is synchronized with one of the spatial harmonics of a radiation field, is widely used in the technique of generation and amplification of centimeter waves. The parallelism between the phenomena occurring in these devices and the Cerenkov effect was emphasized, for example, in [2] (page 59).

It is also interesting to compare our phenomenon with coherent bremsstrahlung of electrons of low and medium energy in a single crystal [5]. Both effects are connected with the interaction between the electron and the spatial harmonics of the fields inside the single crystal: in [5] this field is the atomic field of the lattice, and here it is the radiation field itself. Thus, as in the comparison of ordinary bremsstrahlung with the Vavilov-Cerenkov

effect, the difference lies in the presence or absence of acceleration of the radiating charge. A detailed comparison of Cerenkov radiation with the radiation from a charge moving with acceleration in a spatially-periodic force field (the macroscopic analog of the atomic field of a crystal lattice), which is directly applicable to the question considered here, was made in [6].

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FARADAY EFFECT ON "HOT" ELECTRONS IN SEMICONDUCTORS

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As is well known, heating of an electron gas in a semiconductor can exert a noticeable influence on different characteristics of the latter (electric conductivity, galvanomagnetic and thermomagnetic effects, etc.). In this paper we consider the influence of heating of an electron gas on the Faraday rotation of the plane of polarization of an electromagnetic wave passing through a semiconductor.

We shall characterize the degree of "heating" of the electron gas by means of an electron temperature T , which differs from the lattice temperature T_0 . The possibility of such an approximation was confirmed in several papers (see, for example, [1]). Further, we shall assume that the magnetic field is weak and that the frequency of the electromagnetic wave is sufficiently low to neglect quantum effects. We consider a unipolar semiconductor, in which the carriers are characterized by an isotropic effective mass m^* and the reciprocal momentum relaxation time depends on the carrier velocity like $\tau(v) \sim v^3$. Finally, we confine ourselves to the case of weak heating of the electron gas [$(T - T_0)/T_0 \ll 1$].

The standard solution of Maxwell's equations together with the kinetic equation yields for the Faraday angle (per unit length) the following expression: