CONTRIBUTION TO THE THEORY OF DIAMAGNETISM OF SEMICONDUCTORS IN A QUANTIZING MAGNETIC FIELD

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It is known that in a quantizing magnetic field the scattering of carriers by optical phonons has a resonant character, which is manifest in magnetophonon oscillations [1] and in oscillations of the absorption of light by the free carriers. It was noted in [2] that resonant absorption and emission of phonons leads to singularities in the dispersion part of the carrier spectrum. It is natural to consider the manifestation of these singularities in the diamagnetic susceptibility of the carriers.

We start with the Hamiltonian

$$H = \sum_p \epsilon_p a_p^+ a_p + \sum_q \omega_q (c_q^+ c_q + 1/2) + \sum_{pp'q} G_q (i_{pp'q} a_p^+ c_q a_p + \text{Herm. conj.}) \qquad (1)$$

where a_p^+, a_p, and c_q^+, c_q are creation and annihilation operators, ϵ_p and ω_q are the carrier and phonon energies, respectively, G_q is the interaction constant, p stands for quantities characterizing the state of the carriers in the quantizing magnetic field, q are the wave vectors of the phonons

$$i_{pp'q} = \int d^3x \psi_p^* e^{iqx} \psi_{p'}$$

and ψ_p are the Luttinger and Kohn wave functions [3].

Following [2], we find the carrier spectrum from the poles of the retarded Green's function in the lower half of the energy plane:

$$D_{pp'}(E) = \int e^{iEt} \Theta(t) \,\text{Sp}\, e^{-\beta H} a_p^+(t) a_{p'}(0), \qquad \Theta(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

$$a_p^+(t) = e^{iHt} a_p^+(0) e^{-iHt} \qquad (2)$$

The carrier spectrum has the following form in the second order of the interaction:

$$E = \epsilon_{p'} - \sum \int d^3q \left[\frac{R^2 |L_n^{n'-n}(q_\perp^2)|^2 N_0}{(\epsilon_{n'p'_z+q_z} - \epsilon_{np'_z} - \omega)} + \frac{R^2 |L_n^{n'-n}(q_\perp^2)|^2 (1 + N_0)}{(\epsilon_{n'p'_z-q_z} - \epsilon_{np'_z} + \omega)} \right] \qquad (3)$$

Here ω is the end-point frequency of the optical phonons, N_0 the number of phonons, and

$L_n^{n'-n}(q_\perp^2)$ is a Laguerre polynomial with weight

$$G^2 = R^2/q^2 = \frac{\hbar\omega e^2}{q^2}\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right)$$

Putting $q^2 = \frac{3}{2} q_\perp^2$ in G, we get

$$E = \epsilon_p - \frac{R^2(2m^*)^{1/2}}{2} \sum K_{nn'}\Big[N_0(\Omega(n' - n) - \omega - p_z^2/2m^*)^{-1/2} - (1 + N_0)(\Omega(n' - n) + \omega - p_z^2/2m^*)^{-1/2}\Big], \quad \Omega = eH/m^*c; \; K_{nn'} = \int dq_\perp^2 \, (2/3)\frac{|L_n^{n'-n}|^2}{q_\perp^2} \tag{4}$$

We now calculate the partition function for a gas of carriers with energies renormalized for the interaction with the phonons

$$Z = \frac{2eH}{\hbar^2 c}\sum \int dp_z e^{-\beta E} = Z_0 + \frac{R^2 eH(2m^*)^{1/2}}{\hbar^2 c}\sum_{nn'} \int dp_z\{(1 + N_0)[\Omega(n' - n) + \omega - p_z^2/2m^*] + N_0[\Omega(n' - n) - \omega - p_z^2/2m^*]\}[\exp(-\beta p_z^2/2m^*)]K_{n'n} \tag{5}$$

Z_0 is the partition function without account of the interaction with the phonons. We estimate the integral in (5), using the asymptotic relation (4)

$$\int_a^b \varphi(t) e^{\lambda f(t)} dt \sim e^{\lambda f(t_0)} \sqrt{\frac{\pi}{\lambda}} \sum_n \frac{c_{2n}(2n)!}{4^n n! \lambda^n} \tag{6}$$

where t_0 is the extremal point of $f(t)$. We obtain

$$Z = Z_0 + \frac{R^2 e^H(2m^*)^{3/2}}{\hbar^2 c\beta^{1/2}} \sum_{nn'} K_{nn'} e^{-\beta\Omega(n'+1/2)}\Big\{(1 + N_0)[\Omega(n - n') + \omega]^{-1/2} + N_0[\Omega(n - n') - \omega]^{-1/2}\Big\} \tag{7}$$

We therefore have for the diamagnetic susceptibility

$$\mu = \frac{N}{\beta H}\frac{\partial \ln z}{\partial H} = \mu_0 + \frac{R^2 e(2m^*)^{1/2} N}{\hbar^2 Hc\beta^{3/2} z_0}\sum_{nn'} K_{nn'} e^{-\beta\Omega(n+1/2)}\Big\{(1 + N_0)[\Omega(n' - n) + \omega]^{-1/2} + N_0[\Omega(n' - n) - \omega]^{-1/2} - \frac{\beta\Omega(n + 1/2)}{H}[(1 + N_0)(\Omega(n' - n) + \omega)^{-1/2} + N_0(\Omega(n' - n) - \omega)^{-1/2}] - \frac{\Omega(n' - n)}{H}[(1 + N_0)(\Omega(n' - n) + \omega)^{-3/2} + N_0(\Omega(n' - n) - \omega)^{-3/2}]\Big\} \tag{8}$$

It is obvious that the diamagnetic susceptibility term due to the interaction between the carriers and the optical phonons contains resonant peaks at $\Omega(n' - n) \pm \omega = 0$. Since $\beta\Omega > 1$, only small n are important in (8), so that the terms containing $(n' - n)\Omega - \omega$ come into play when $n = 0$. It is obvious that the resonant absorption of the optical phonons leads to peaks also when ω is an integer multiple of Ω. However, owing to the exponential decrease of N_0, for each succeeding peak with increasing $\beta\omega$, the most essential contribution will be made by the resonant absorption of the phonon at the peak $\omega = \Omega$. In the terms corresponding to resonant emission, the singularities will be observed at $n > n'$; the presence of an exponential factor will again separate a peak at $n' = 0$ and $n = 1$. We can thus expect an essential non-monotonicity to appear in the diamagnetic susceptibility only when $\omega = \Omega$. A complete calculation, with account of attenuation in the carrier spectrum, which determines the shape of the resonance lines, entails no difficulty of principal nature, but is technically very cumbersome. The calculation presented determines correctly the position of the resonant peaks.

[1] V. L. Gurevich and Yu. A. Firsov, JETP 47, 734 (1964), Soviet Phys. JETP 20, 489 (1965).

[2] Z. I. Uritskii and G. V. Shuster, JETP 49, No. 7 (1965) (in press).

[3] J. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955).

[4] M. A. Lavrent'ev and B. V. Shabat, Metody teorii funktsii kompleksnykh peremennykh (Methods of Complex Variable Theory), Fizmatgiz, 1958, p. 449.

ELECTROPRODUCTION OF ISOBAR $N_{3/2}(1238)$ IN THE SU(6) SYMMETRY SCHEME

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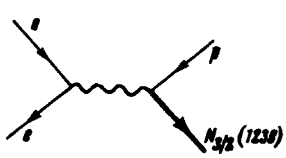
If the magnetic moment of the proton is known, the theory of SU(6) symmetry [1-3] makes it possible to calculate not only the magnetic moments of all terms of the 56-plet, but also the magnetic moments of the transitions between various components of this multiplet. Let us consider the reaction $e + p \to p + N_{3/2}(1238)$ (see the figure). Since both the proton and the $N_{3/2}(1238)$ belong to a representation with total orbital angular momentum $L = 0$, the matrix element of the quadrupole moment is equal to zero. Consequently, the transition will be



magnetic-dipole. In SU(6) theory the non-diagonal matrix element of the magnetic moment is equal to [4]