ACOUSTIC OSCILLATIONS AND PROPAGATION OF ELASTIC WAVES IN A LATTICE WITH HEAVY INTERSTITIAL IMPURITIES

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When considering oscillations of lattices with impurity centers, it is usually assumed that the corresponding point-like defect is a substitutional impurity, i.e., it is located at a crystal lattice site. We wish to call attention to the fact that the presence of interstitial impurities in the lattice, which increase the number of degrees of freedom of the crystal, leads to a peculiar variation of the lattice vibrations. Such a situation was first noted by Wagner [1,2] in an analysis of molecular impurities in a lattice.

Following Lifshitz [3], we confine ourselves to a simple model of a lattice possessing only one vibrational mode, and denote by $\chi(\underset{\sim}{r})$ the amplitudes of the oscillations of the atom at the site $\underset{\sim}{r}$, and by ψ_j the amplitude of the impurity atom vibration at the point $\underset{\sim}{R}_j$. Then the equations of the stationary lattice vibrations of frequency ω should have in the harmonic approximation the form

$$\begin{cases} \omega^2\chi(\underset{\sim}{r}) - \sum\limits_{\underset{\sim}{r}'} L(\underset{\sim}{r} - \underset{\sim}{r}')\chi(\underset{\sim}{r}') = \sum\limits_j \left\{ \sum\limits_{\underset{\sim}{r}'} U(\underset{\sim}{r} - \underset{\sim}{R}_j, \underset{\sim}{r}' - \underset{\sim}{R}_j)\chi(\underset{\sim}{r}') + A(\underset{\sim}{r} - \underset{\sim}{R}_j)\psi_j \right\} \\ \omega^2 M \psi_j = M \sum\limits_{\underset{\sim}{r}} A(\underset{\sim}{r} - \underset{\sim}{R}_j)\chi(\underset{\sim}{r}) + \alpha_0^2 \psi_j \end{cases} \qquad (1)$$

where $\hat{L}$ is the matrix of the elastic constants of the ideal lattice, M is the mass of the impurity atom, α_0^2 is the constant of elastic coupling between the impurity atom and the lattice, and matrices $\hat{U}$ and $\hat{A}$ describe the corresponding perturbations of the elastic constants.

If we use the conditions that interrelate the perturbation matrices [3], then we can eliminate the amplitudes ψ_j from (1).

To simplify the analysis we assume, following [3], that the matrix $\hat{U}$ is multiplicative $U(\underset{\sim}{\xi}, \underset{\sim}{\xi}') = U_0 f(\underset{\sim}{\xi}) f(\underset{\sim}{\xi}')$. Then the aforementioned relations between perturbation matrices imply the following properties:

$$A(\underset{\sim}{\xi}) = -\omega_0\sqrt{U_0}\, f(\underset{\sim}{\xi}), \qquad \sum_{\underset{\sim}{r}} f(\underset{\sim}{r} - \underset{\sim}{R}) = \omega_0/\sqrt{U_0}, \qquad \omega_0^2 = \alpha_0^2/M, \quad U_0 > 0 \qquad (2)$$

and the equation for $\chi(\underset{\sim}{r})$ reduces, after elimination of the amplitudes ψ_j, to

$$\omega^2 \chi(\underset{\sim}{r}) - \sum_{\underset{\sim}{r}'} L(\underset{\sim}{r} - \underset{\sim}{r}')\chi(\underset{\sim}{r}') = [(\omega^2 U_0)/(\omega^2 - \omega_0^2)] \sum_j f(\underset{\sim}{r} - \underset{\sim}{R}_j) \sum_{\underset{\sim}{r}'} f(\underset{\sim}{r}' - \underset{\sim}{R}_j)\chi(\underset{\sim}{r}') \qquad (3)$$

For long waves, in the description of which the coordinates of the atoms $\underset{\sim}{r}$ can be regarded as running through a continuous set of values, we put $f(\underset{\sim}{\xi}) = \delta(\underset{\sim}{\xi})$, where $\delta(\underset{\sim}{\xi})$ is a delta function (then $U_0 = \omega_0^2$).

In the case when the frequency ω_0 lies above the continuous-spectrum band of the acoustic vibrations, Eq. (3) is of the same type as investigated by Lifshitz [3]. However, if ω_0 falls into continuous-spectrum band (large masses M), then the perturbation operator in (3) has a resonant denominator, the presence of which is quite significant in those cases when ω_0 lies in the vicinity of the lower or upper edge of the continuous spectrum [4]. To explain the role of the resonant denominator, we assume that $\omega_0 \ll \omega_D$, where ω_D is the Debye frequency, i.e., we consider the limiting case of very large masses M and confine ourselves to the long-wave approximation, which simplifies (3).

We shall assume that the concentration of the impurities is sufficiently small and their distribution is on the average homogeneous. Then for elastic oscillations whose wavelengths greatly exceed the mean distances between impurities, we can speak of waves that are "plane in the mean," putting $\chi(\underset{\sim}{R}) = \chi \exp(i\underset{\sim}{k}\cdot\underset{\sim}{R})$. The dispersion law for these waves, in the approximation that is linear in the concentration, can be obtained from the following equation (see Sec. 5 of [3]):

$$\omega_0^2(\underset{\sim}{k}) = \omega^2 n^2(\omega)$$

$$n^2(\omega) = 1 - c\Lambda(\omega)\left\{1 + \Lambda(\omega)\int \omega^2\nu(z)\,dz/(z^2 - \omega^2)\right\}^{-1}$$

where $\omega = \omega_0(\underset{\sim}{k})$ is the dispersion law for an ideal lattice (in the isotropic approximation for long waves, $\omega_0(\underset{\sim}{k}) = sk$, where s is the speed of sound, c is the impurity concentration, $\Lambda(\omega) = \omega_0^2/(\omega^2 - \omega_0^2)$, and $\nu(\omega)$ is the spectral density of the frequencies of the ideal crystal.

When $\omega \ll \omega_D$ we have

$$\int[\omega^2\nu(z)dz/(z^2 - \omega^2)] \cong (\omega/\omega_{cr})^2 + i(\pi/2)\omega\nu(\omega), \qquad (1/\omega_{cr}^2) = \int[\nu(z)dz]/z^2$$

where, as obvious from the definition of ω_D, we have $\omega_{cr} \sim \omega_D$.

We put $n(\omega) = n_0(\omega) + i\varkappa(\omega)$; then n_0 and $\varkappa$ are determined from the relations

$$\begin{aligned} n_0^2 - \varkappa^2 &= 1 - c\{[(\omega^2 - \omega_{imp}^2)\,\omega^2{}_0] / [(\omega^2 - \omega_{imp}^2)^2 + \Gamma^2(\omega)]\} \\ 2n_0\varkappa &= c\{[\omega_0^2\Gamma(\omega)] / [(\omega^2 - \omega_{imp}^2)^2 + \Gamma^2(\omega)]\}; \qquad \Gamma(\omega) = (\pi/2)\,\omega_0^2\,\omega\,\nu(\omega) \end{aligned} \tag{4}$$

where ω_{imp} is the renormalized frequency of the proper oscillations of the impurity: $\omega_{imp} = \omega_0[1 - \omega_0/\omega_{cr}]$. When $\omega \ll \omega_D$ we have in order of magnitude $\nu(\omega) \sim \omega^2/\omega_D^3$ and $\Gamma(\omega) \sim \omega_0^2(\omega/\omega_D)^3 \ll \omega_0^2$.

The meaning of the quantities n_0 and $\varkappa$ is known: in the isotropic approximation n_0 determines the renormalization of the speed of sound in the lattice with impurities: $s_n(\omega) = s/n_0(\omega)$ and $\varkappa$ is related to the coefficient γ of elastic-wave absorption in the crystal by $\gamma(\omega) = \omega\ (\omega)/s$.

From (4) it follws that, as in the case of quasi-local oscillations [4], the absorption of the elastic waves has a clearly pronounced resonant character. The resonant frequency

$\omega = \omega_{imp}$ divides as it were the frequency spectrum into two regions, in the first of which ($\omega \ll \omega_{imp}$) the vibrations of the crystal lattice with the defect are ordinary acoustic vibrations. The second region of frequencies ($\omega \gg \omega_{imp}$) corresponds to vibrations for which the defects remain practically stationary during the course of oscillation of the main atoms of the lattice. These vibrations recall optical vibrations of a complex lattice.

We note that the ratio of the attenuation length of the wave at resonance to its length is on the order of $c\omega_0^2/\Gamma(\omega_0)$. At an impurity concentration $c \sim \Gamma(\omega_0)/\omega_0^2 \sim (\omega_0/\omega_D)^3$ a plane elastic wave should attenuate at resonant frequency over a distance on the order of the wavelength. A crystal lattice with such an impurity concentration would be practically "opaque" to these waves in the narrow frequency interval $\Delta\omega \sim c\omega_0$ near $\omega = \omega_{imp}$ and its frequency spectrum would be separated into acoustic and "optical" bands with a narrow gap between them.

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CROSS SECTION FOR THE ABSORPTION OF γ QUANTA BY OXYGEN NUCLEI IN THE ENERGY INTERVAL 13.5 - 22 MeV

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The cross section for the total absorption of γ quanta by O^{16} nuclei was previously measured by the absorption method in the 13 - 27 MeV interval [1,2]. In the present paper we present results of more accurate measurements, by the same procedure, of the cross section for the absorption of γ quanta by O^{16} nuclei in the 13.5 - 22 MeV interval.

The measurements were made with the 260-MeV synchrotron of the Physics Institute of the Academy of Sciences. The absorber (distilled water) had a thickness 100 g/cm^2. To increase the efficiency of the method, the γ rays were detected with a 9-channel paired magnetic spectrometer, described in detail in [3]. The resolving power of the spectrometer in the investigated energy interval was ~ 100 keV (10 MeV - ~ 70 keV; 20 MeV - ~ 110 keV).

Figure 1 shows the nuclear absorption cross section σ_{nuc} obtained after subtracting the cross sections for Compton scattering and pair production in the field of the nucleus and of the electrons. The subtracted summary cross section of these processes was normalized in the 13.5 - 15.6 MeV interval [1].

The absolute scale of the nuclear cross section was determined accurate to ± 1 mb. The errors presented are mean-square. The energy scale was corrected for the finite resolving power of the spectrometer.