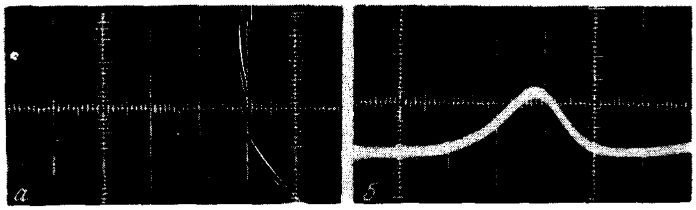
located between the neodymium rod and the mirror with $R_1 = 99\%$, was $\sim 20\%$ for $\lambda = 1.06\ \mu$.

Under these conditions, we obtained a single pulse of duration $\sim$ 10 nsec (Fig. 1a). At 3,000 J pump energy the pulse power was approximately 50 MW, and a spark was observed at the focus of a lens of f = 500 mm. With increasing effective length of the cavity, the duration increased nonlinearly, reaching $\sim$ 330 nsec at L_{eff} = = 300 cm (Fig. 1b).



Output signal oscillograms for L_{eff} = 55 cm (a, time scale 20 nsec/cm) and 300 cm (b, time scale 250 nsec/cm).

It must be noted that at an effective cavity length L_{eff} = 55 cm, a pulse duration of $\sim$ 10 nsec corresponds to the time necessary for the quantum to travel five times between the rotating mirrors. Inasmuch as in our case the threshold was exceeded relatively little, the single-pulse duration was practically close to the limit and was determined by the cavity and not by the shutter. At the same time, the results obtained indicate that the on-time of our shutter was lower than 10 nsec.

To obtain a single pulse of even shorter duration it is obviously necessary to reduce the effective length of the cavity and to increase the initial inverse population of the metastable level.

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[1] A. M. Prokhorov, Radiotekhnika i elektronika 8, 1073 (1963).
[2] A. L. Mikaelyan and Yu. G. Turkov, ibid 9, 743 (1964).
[3] Kafalas, Masters, and Murray, J. Appl. Phys. 35, 2349 (1964).
[4] B. H. Soffer, J. Appl. Phys. 35, 2551 (1964).
[5] Basov, Ambartsumyan, Zuev, Kryukov, and Stoilov, JETP 47, 1595 (1964), Soviet Phys. JETP 20, 1072 (1965).
[6] B. H. Soffer and R. H. Hoskins, Nature 204, 276 (1964).
[7] Malyshev, Markin, and Petrov, JETP, Letters to Editor 1, No. 3, 49 (1965), translation p. 99.

1) A pulse of duration $\sim$ 15 nsec was obtained in [5] using Q-modulation of a Kerr cell.

CASIMIR OPERATORS FOR THE UNITARY GROUP

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A group-theoretical trend is now actively pursued in the theory of elementary particles, with the U(n) and SU(n) groups used most successfully to describe the symmetries of elementary particles. It is of interest in this connection to determine all the invariant operators that can be formed from the generators of a group. Although this problem was already considered earlier [1-3], there are no published explicit expressions for the eigenvalues of invariant operators of arbitrary order. We present a solution of this problem.

The generators of the groups U(n) and SU(n) satisfy the commutation relations

$$[A^i_j, A^k_\ell] = \delta^k_j A^i_\ell - \delta^i_\ell A^k_j \qquad (1)$$

and in the case of the SU(n) group the condition $\Sigma^n_{i=1} A^i_i = 0$ is satisfied. An invariant operator (or Casimir operator) of arbitrary order p is of the form

$$C_p = \sum_{i_1,\ldots,i_p=1}^{n} \left[A^{i_1}_{\ell_2} A^{i_2}_{\ell_3} \ldots A^{i_p}_{i_1}\right] \qquad (2)$$

Using (1) and the commutation relations between A^i_j and an arbitrary tensor operator T^k_ℓ, we obtain from (2):1)

$$C_p(f_1, \ldots, f_n) = \sum_{i,j=1}^{n} (a^p)_{ij} \qquad (3)$$

where $C_p(f_1, \ldots, f_n)$ is the eigenvalue of the operator (2) for the irreducible representation specified by the Young tableau $\{f_1, f_2, \ldots, f_n\}$; f_i is the number of boxes in the i-th row, $f_1 \geq f_2 \geq \ldots \geq f_n \geq 0$, with $f_n = 0$ for the SU(n) group. The matrix a is of the form

$$a_{ij} = (m_i + n - i)\delta_{ij} - \theta_{ij}, \quad \theta_{ij} = \begin{cases} 1 & i < j \\ 0 & i \geq j \end{cases} \qquad (4)$$

Here $m_i = f_i$ for the U(n) group and $m_i = f_i - f/n$ for the SU(n) group; $f = f_1 + f_2 + \ldots + f_n$.

From (3) follows a connection between the Casimir operators for the U(n) and SU(n) groups:

$$C^{(U)}_p = \sum_{\alpha=0}^{p} \frac{p!}{\alpha!(p-\alpha)!} \left(\frac{f}{n}\right)^{p-\alpha} C^{(SU)}_\alpha$$

$$C^{(SU)}_p = \sum_{\alpha=0}^{p} \frac{p!}{\alpha!(p-\alpha)!} \left(-\frac{f}{n}\right)^{p-\alpha} C^{(U)}_\alpha$$

where

$$C^{(U)}_0 = C^{(SU)}_0 = n, \; C^{(U)}_1 = f, \; C^{(SU)}_1 = 0 \qquad (5)$$

Introducing $L_i = m_i + n - i$, we can transform (3) into

$$C_p = \sum_{i=1}^{n} L^p_i - \sum_{\alpha+\beta=p-1} \sum_{i<j} L^\alpha_i L^\beta_j + \ldots + (-)^p \sum_{i_1<i_2<\ldots<i_p} 1. \qquad (6)$$

Each term of (6) is a symmetrical function of $L_1, L_2, \ldots, L_n$, and can be expressed in terms of standard functions $S_\alpha = \Sigma^n_{i=1}[L^\alpha_i - (n - i)^\alpha]$. We present the corresponding formulas for the first six Casimir operators of the SU(n) group:

$$C_1 = 0, \; C_2 = S_2, \; C_3 = S_3 - (n - \tfrac{3}{2})S_2, \; C_4 = S_4 - (n - 2)S_3 - \tfrac{1}{2}(3n - 4)S_2,$$

$$C_5 = S_5 - (n - \tfrac{5}{2})S_4 - \tfrac{1}{2}S^2_2 - \tfrac{2}{3}(3n - 5)S_3 - \tfrac{1}{2}(4n - 5)S_2, \quad C_6 = S_6 - \qquad (7)$$

$$- (n - 3)S_5 - S_3S_2 - \tfrac{5}{2}(n - 2)S_4 + \tfrac{1}{2}(n - 3)S^2_2 - \tfrac{5}{3}(2n - 3)S_3 - \tfrac{1}{2}(5n - 6)S_2.$$

We note that the value of the C_2 coincides with that obtained previously by Racah [5], while those of C_3 and C_4 coincide with the values obtained in [3]. In the case of the SU(6) group the operators C_p of (7) give the complete set of independent invariants that can be constructed from A^i_j. From (7) we obtain as a particular case the formulas for the Casimir operators of the SU(3) group

$$C_2 = \frac{2}{3}(p^2 + q^2 + pq + 3p + 3q),$$
$$C_3 = \frac{1}{9}(p - q)[(p + 2q)(2p + q) + 9(p + q + 1)] + \frac{3}{2}C_2 \tag{8}$$

which have already been used in calculations [6].

The simplest representations of the U(n) and SU(n) groups are the completely symmetrical representations $\{f\}$ and the completely antisymmetrical representations $\{1^k\}$. With the aid of (3) we can obtain the eigenvalues of all the operators C_p for these representations:

$$C_p(\{f\}) = f(f + n - 1)^{p-1}$$
$$C_p(\{1^k\}) = k(n - k + 1)^{p-1} \tag{9}$$

These formulas pertain to the U(n) group; the values of C_p for the SU(n) group can be obtained from them by taking (5) into account. A more detailed exposition of the results will be published separately.

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[1] I. M. Gel'fand, Matem. sb. (Mathematics Collection) 26, 103 (1950).
[2] G. E. Baird and L. C. Biedenharn, J. Math. Phys. 4, 436, 1449 (1963).
[3] M. Umezawa, Nucl. Phys. 48, 111 (1963); 53, 54 (1964); 57, 65 (1964).
[4] M. Micu, Nucl. Phys. 60, 353 (1964).
[5] G. Racah, Group Theory and Spectroscopy, Lecture Notes, Princeton, 1951.
[6] S. P. Rosen, Phys. Rev. 135B, 1041 (1964).

1) The method used by us to obtain formula (3) makes use of some ideas taken from [4].

OBSERVATION OF A FAST PHOTOIONIZATION AUREOLE AND OF A CONCENTRATED LONG LIVED IONIZATION CLOUD DUE TO A SHOCK WAVE FROM A SPARK IN A LASER BEAM

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The vigorous release of energy in a gas by a concentrated laser beam ("optical spark") [1-3] should be accompanied by intense ionization induced in the gas by the radiation photons due to the high temperature heating and the resultant strong shock wave. We have observed a rapidly arising aureole of ionization due to the photons, anticipating the shock wave. An important role in its production is apparently played by multiple ionization, absorption,