trace), taken with a sweep of 80 μsec. We see that the maximum overlap is obtained after ~ 5 μsec, a time commensurate with the time of passage of the shock wave, which envelopes the aureole after several microseconds. Noticeable overlap is observed also at times that are insufficient for the shock wave to cover the effective transverse dimensions of the overap section. In the initial stage of the overlap, the growth of a signal proportional to the interaction cross section is faster than the time growth of the cross section for scattering by the shock wave. For the shock wave the radius is $\sim t^{2/5}$ and the cross section is $\sigma \sim r^6 \sim t^{12/5}$ for $r < \lambda\!\!\!^{-}$ and $\sigma \sim r^2 \sim t^{4/5}$ for $r \gtrsim \lambda\!\!\!^{-}$, while experiment yields a much steeper growth of the signal during the initial stage. The presence of a fast ionization aureole, produced within a fraction of a microsecond and anticipating the shock wave, was clearly recorded by means of special probes.

We note that with increasing microwave wavelength the effective overlap and scattering by the fast aureole should increase, since the aureole does not have too high a concentration, while the critical concentration is proportional to the square of the frequency of the wave. Concentrated ionization and heating of the gas by photoionization and by the shock wave in the vicinity of the spark lengthens the plasma lifetime appreciably compared with normal temperatures. The prolonged existence of the ionization aureole around the spark explains the long-lived disturbance of the magnetic field by the spark plasma, previously observed by the authors [4].

The results obtained for the interaction between microwaves and an optical spark, and an investigation of the ionization aureole, can be used to attempt to transfer energy to the strongly dissipating spark plasma from rapidly varying intense electromagnetic waves, induction fields, or light beams, and also to use the ionizing rays from a laser as reflectors, guidance systems, and radio antennas.

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INCOMPATIBILITY OF RELATIVIZED SU(6) SYMMETRY WITH UNITARITY

B. V. Geshkenbein, B. L. Ioffe, M. S. Marinov, and V. I. Roginskii
Nuclear Physics Division, USSR Academy of Sciences
Submitted 10 May 1965

1. Many recent papers are devoted to a relativistic generalization of SU(6) symmetry, and are aimed at constructing S-matrix elements that are invariant relative to the Lorentz group ($\mathfrak{L}$) of the SU(3) group and SU(6)-invariant in the static limit (for example, [1,2]).

Such theories establish certain linear relations between the invariant amplitudes for arbitrary momenta. On the other hand, no such relations could be obtained so far on the basis of the invariance with respect to some exact group (and not a "self breaking" group, as in the existing theories), which would include the direct product $\mathcal{L} \times SU(3)$ as a nontrivial subgroup. The reason is that the existence of such a general group is in conflict with the free equations of motion. Relations that have no group origin, however, generally speaking contradict unitarity. The purpose of the present note is to clarify the concrete structure and the degree of this contradiction by means of simple examples. We consider the amplitudes of scattering of the singlet state by a quark and of scattering of a quark by a quark in the $\tilde{U}(12)$ scheme [1]. The difficulties encountered are apparently characteristic of any relativization of SU(6).

2. Let us consider the amplitude of scattering of a spinless particle, which is a unitary singlet, by a quark. This amplitude is of the form $A + B(\hat{q}_1 + \hat{q}_2)/2$. According to the $\tilde{U}(12)$ scheme

$$B = 0 \tag{1}$$

By virtue of (1), elastic unitarity imposes two real conditions on one complex function A. Let us ascertain whether this system of equations has a solution. To this end we expand the scattering amplitudes in partial waves (see, for example, [3]). Transforming these expansions under the assumption that they are convergent, we can write condition (1) in the form of an assembly of equations for the partial waves:

$$f^{+}_{\ell-1} - f^{-}_{\ell+1} + \kappa(f^{-}_{\ell} - f^{+}_{\ell}) = 0 \qquad \ell = 1, 2, \ldots, \tag{2}$$

where

$$f^{\pm}_{\ell} = p^{-1}\exp(i\delta^{\pm}_{\ell})\sin\delta^{\pm}_{\ell}, \qquad \kappa = (\epsilon + m)/(\epsilon - m), \tag{3}$$

and $\delta^{\pm}_{\ell}$ are the scattering phases with $j = \ell \pm 1/2$ respectively.

In the energy region where the elastic unitarity condition is valid, the phases $\delta^{\pm}_{\ell}$ are real, making it possible to solve (2), for specified ℓ, with respect to δ^{+}_{ℓ} and $\delta^{-}_{\ell+1}$. Equation (2) has two solutions:

$$\begin{aligned} &\text{I.} \quad \delta^{-}_{\ell+1} = \delta^{-}_{\ell} + \delta^{+}_{\ell} - \delta^{+}_{\ell-1} && \text{II.} \quad \delta^{+}_{\ell} = \delta^{-}_{\ell} \\ &\qquad \sin(\delta^{-}_{\ell+1} - \delta^{+}_{\ell-1}) = \kappa\sin(\delta^{-}_{\ell} - \delta^{+}_{\ell}) && \qquad \delta^{-}_{\ell+1} = \delta^{+}_{\ell-1} \end{aligned} \tag{4}$$

Choosing for each value of ℓ any of the solutions I or II, we can express all the phases $\delta^{\pm}_{\ell}$ in terms of only two phases, namely δ_0 and δ^{+}_{1}. It is easy to show that for an arbitrary alteration of solutions I and II, the phases do not tend to zero as $\ell \to \infty$. For example, by choosing solutions of type I for all values of ℓ we get

$$\begin{gathered} \delta^{+}_{\ell} = \ell\delta^{+}_{1} - (\ell - 1)\delta_0, \qquad \delta^{-}_{\ell} = (\ell - 1)\delta^{+}_{1} - (\ell - 2)\delta^{+}_{0} + \alpha \\ \tan\alpha = (\epsilon/m)\tan(\delta_0 - \delta^{+}_{1}) \end{gathered} \tag{5}$$

A convergent series is obtained only when all $\delta^{\pm}_{\ell} = 0$.

Even if we assume that the $\tilde{U}(12)$ scheme is broken for large l, something that could ensure convergence, the solution of (4) still remains without physical meaning, since these equations contradict the threshold behavior of the phases $\delta_e^{\pm} \sim p^{2l+1}$. It must be noted that relations (2) lead to paradoxical conclusions even if we forego the requirement of elastic unitarity. Indeed, it follows from (2) that if resonance exists, then it should be present in all the partial waves.

Let us find further with the aid of (2) the jump $\Delta f_e^{\pm}$ of the partial amplitude near the threshold of production of n spinless particles (such that the products of the intrinsic parities of the particles are the same at the beginning and at the end). Using the threshold behavior of $\Delta f_e^{\pm}$ and the positiveness of $\mathrm{Im}\Delta f_e^{\pm}$, we can easily show by virtue of (2) that $\Delta f_e^{\pm} \equiv 0$.[1)]

3. In the $\tilde{U}(12)$ scheme the amplitude for the elastic scattering of a quark by a quark is of the form

$$\Phi_1 \bar{\psi}^A(p_1')\psi_A(p_1)\cdot\bar{\psi}^B(p_2')\psi_B(p_2) + \Phi_2 \bar{\psi}^A(p_2')\psi_A(p_1)\cdot\bar{\psi}^B(p_1')\psi_B(p_2) \qquad (6)$$

where Φ_1 and Φ_2 are invariant functions. Let us consider the elastic scattering of two identical quarks. Using the notation of [4], we express the amplitudes F_1, F_2, F_3, F_4, and F_5 preceding the Fermi variant in terms of Φ_1 and Φ_2:

$$F_1 = \frac{1}{2}(\Phi_1 + \Phi_2), \quad F_2 = F_4 = \frac{1}{4}(\Phi_1 - \Phi_2), \quad F_3 = F_5 = 0 \qquad (7)$$

The elastic unitarity condition relates the two complex functions Φ_1 and Φ_2 (i.e., four real functions) by means of five real equations. This system of equations can have a nontrivial solution only if the equations are dependent. Using the expansion of the scattering amplitudes in partial waves, we can show that the equations are independent and consequently the scattering amplitude vanishes. Even without the requirement of elastic unitarity, as in the preceding section, the existence of a resonance in one partial wave implies the presence of resonances in all the partial waves.

4. The examples just considered are simple but are not realistic, since they presuppose the existence of quarks. We can hardly expect, however, to reconcile the $\tilde{U}(12)$ scheme with unitarity in more realistic cases. For example, in the $\tilde{U}(12)$ scheme the processes $\underline{56} + \underline{56} \to \underline{56} + \underline{56}$ or $\underline{56} + \underline{35} \to \underline{56} + \underline{35}$ are described by only four amplitudes each, in lieu of the several hundred amplitudes needed in ordinary theory. It is little likely that a unitary S matrix can withstand such coercion.

It is interesting to ascertain how a symmetrical scheme must be broken in order for the theory to become unitary, and whether the symmetry is not reduced thereby to $\mathcal{L} \times SU(3)$.

After the completion of this paper, the authors received a preprint by Beg and Pais [5], who considered the problem of unitarity in relativized SU(6) symmetry and noted the associated difficulties.

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"SHADOW UNIVERSE" AND THE NEUTRINO EXPERIMENT

L. B. Okun' and I. Ya. Pomeranchuk
Division of Nuclear Physics, USSR Academy of Sciences
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To explain the θ decay of the long-lived K^0 meson [1-3], Nishijima and Saffouri recently proposed the "shadow universe" hypothesis [4], according to which there exists besides our universe U_a also a universe U_b. To each particle a from U_a corresponds a particle b from U_b. Strong and electromagnetic interactions are the same within each of the universes, but there are none between particles from different universes. Weak interaction exists between a particles, and also between a and b, but there is no weak interaction between particles b, so that if there were no $a \longleftrightarrow b$ transitions then K^b mesons would be stable. This form of weak interaction prohibits the unobservable decays of K^a mesons into b particles (for example, π^b mesons).

In the "shadow universe" model, the K_1^0 mesons are degenerate: there exists K_1^a with $\Gamma_a = 10^{10}\ \text{sec}^{-1}$ and K_1^b with $\Gamma_b = 0$. Owing to $K_1^a \longleftrightarrow K_1^b$ in vacuum transitions, two diagonal states K_1 and K_3 are produced

$$
\begin{aligned}
iK_a &= \lambda_a K_a + \lambda K_b \\
iK_b &= \lambda K_a + \lambda_b K_b \\
\lambda_a &= \mu_a - i\Gamma_a/2 \\
\lambda_b &= \mu_b
\end{aligned}
\tag{1}
$$

The frequencies of the diagonal states are

$$\lambda_{1,3} = \frac{\lambda_a + \lambda_b}{2} \pm \sqrt{\frac{(\lambda_a - \lambda_b)^2}{4} + \lambda^2} \tag{2}$$

If we assume that $\beta = \lambda/(\lambda_a - \lambda_b) \ll 1$, then

$$
\begin{aligned}
\lambda_1 &= \mu_1 - i\Gamma_1/2 = \lambda_a + \beta^2(\lambda_a - \lambda_b) \\
\lambda_3 &= \mu_3 - i\Gamma_3/2 = \lambda_b - \beta^2(\lambda_a - \lambda_b)
\end{aligned}
\tag{3}
$$