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# INTERACTION OF THE ALTERNATING JOSEPHSON CURRENT WITH RESONANT MODES IN A SUPERCONDUCTING TUNNEL STRUCTURE 1)

I. M. Dmitrenko, I. K. Yanson, and V. M. Svistunov
Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences
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It was noted earlier [1,2] that the voltage-current characteristics of superconducting-film tunnel structures, which clearly display the Josephson effect, also exhibit small steps characterized by the fact that the change of the current through the tunnel junction, a change usually determined by the measuring circuit, occurs at almost constant voltage on the junction, $V \neq 0$. We have shown [3] that this is accompanied by emission of photons of frequency $\omega = 2eV/\hbar$, corresponding to the frequency of the alternating Josephson supercurrent [4]. In the present note we propose a simple model, in which the steps result from excitation of resonant electromagnetic oscillations in a tunnel structure when alternating Josephson current flows between the films. We present experimental data confirming these notions.

Let us consider the conditions for the propagation of electromagnetic waves in a layer of oxide between superconducting tin films. In such a system there can propagate slowed-down transverse electromagnetic waves [5] with phase velocity $\bar{c} = c\sqrt{[\ell/\epsilon d]}$, where c is the velocity of light, $\ell$ the thickness of the oxide, $\epsilon$ the dielectric constant of the oxide, and $d = 2\lambda_L + \ell$ ($\lambda_L$ is the London depth of penetration). Since the wave resistance of this strip line is very small [4,5], strong reflection of the wave will occur on the boundary, and resonant modes with sufficiently high Q will be able to exist in the bounded system. The condition for the resonance of the electromagnetic waves in the region forming the tunnel junction between the films is written in the form

$$n\frac{\lambda_p^{(n)}}{2} = W \qquad (n = 1, 2, 3, \ldots) \qquad (1)$$

where $\lambda_p^{(n)}$ is the n-th resonant wavelength in the strip resonator and W is the dimension of the rectangular resonator along which resonance occurs. Using the Josephson frequency ratio, the expression for $\bar{c}$, and the temperature dependence $\lambda_L(T) = \lambda_L(0)\ [1 - T/T_k)^4]^{-1/2}$, we can obtain the entire set of voltage $V_p^{(n)}$ as arranged on the voltage axis as functions of the temperature:

$$V_p^{(n)}(T) = \frac{h}{2e}\sqrt{\frac{\ell}{\epsilon d}}\ \frac{\pi c n}{W}\ [1 - (T/T_k)^4]^{-1/4} \tag{2}$$

We thus treat the occurrence of a step as a consequence of excitation of resonant oscillations of electromagnetic waves in the tunnel structure by the alternating Josephson current.

We have investigated experimentally tunnel structures of the type Sn-I-Sn (I = insulator 10 - 20 Å thick), similar to those described in [2,3], as well as more complicated ones (see Fig. 1).

The observed maximum values of the direct Josephson current were 0.8 - 0.95 of the theoretically predicted value [8,9]. The dependence of the critical value of the direct Josephson current is well described by the theoretical formulas of [4,8]. The experimental results show that for each tunnel structure there is a discrete set of voltages $V_p^{(n)}$, at which steps appear when a constant magnetic field on the order of 1 Oe is applied parallel to a film of width W; the multiplicity of the positions of the steps is clearly pronounced $V_p^{(n)} = nV_p^{(1)}$, where n = 1, 2, 3, .... Equality of the phase velocities is satisfied for the 13th step. The voltage at which the first step $V_p^{(1)}$ is observed is inversely proportional to W, with W varying by a factor of several times. The position of the step on the V axis is practically independent of the magnetic field. Figure 1 shows the voltage-current characteristics of one of the tunnel junctions of the structure shown in the insert. The area of the transition is S = 1.25 x 0.36 mm, and T = 1.6°K. The widths of the lower films of the sample in the insert were 0.23, 1.25, and 0.45 mm respectively, and the width of the upper film was 0.36 mm. The numerical values of $V_p^{(n)}$ are in good agreement with those calculated by formula (2), if we assume that $\ell$ = 20 Å and $\epsilon$ = 9. The measured temperature dependence of the position of the first step (Fig. 2) is in good agreement with formula (2).

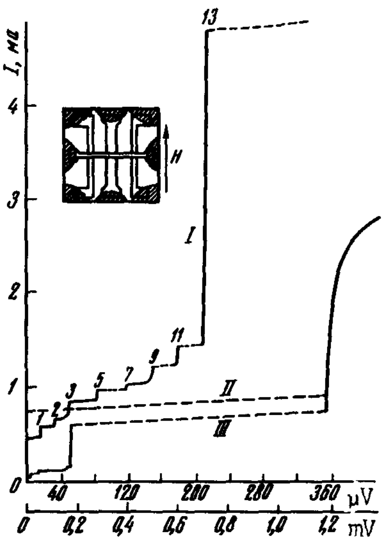

Fig. 1. Voltage-current characteristics of Sn-I-Sn tunnel junction: 1 - initial section (H = 1.12 Oe); 1, 2, 3, ... - numbers of steps; II - total volt-ampere characteristic (H = 0); III - the same (H = 1 Oe). The current scale for II and III should be increased by a factor of 10.

For maximum interaction of the current wave with the electromagnetic wave, it is necessary, in addition to the condition that the frequency of the alternating Josephson current be constant, that their velocities be qual. This leads to the relation [6]

$$V_p^{(n)} = \alpha\sqrt{\frac{\ell}{\epsilon d}}\ H_{max} \tag{3}$$

which yields the value of the constant magnetic field at which the interaction is maximal. The

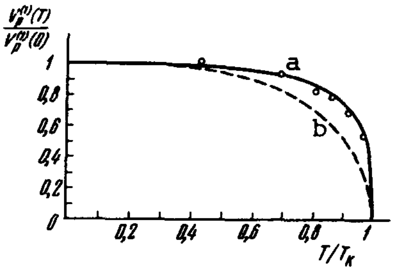

Fig. 2. a - Relative change in the position of the first step as a function of the relative temperature. Continuous line - the function $[1 - (T/T_k)^4]^{1/4}$; the cricles denote the experimental points; b - plot of $\Delta(T)$.

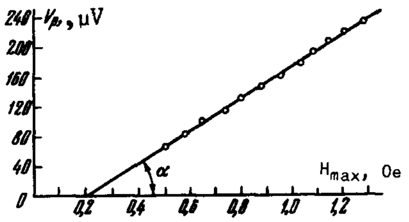

Fig. 3. Voltage of the n-th step for which relation (3) is satisfied vs. the constant magnetic field.

experimentally observed steps constitute the dc component of the Josephson current, a result of self-modulation [6,7], and therefore the height of the step depends on the magnetic field. In a magnetic field of value $H_{max}$ for the n-th step, the height of the latter has a maximum. The experimental plot of $V_p^{(n)}$ vs $H_{max}$ is a straight line (Fig. 3), thus confirming Eq. (3). The fact that the straight line does not pass through the origin is connected with the shift of the principal maximum of the plot of the critical direct Josephson current against the magnetic field [2]. Since relations (2) and (3) are independent (relation (2) holds also in the nonresonant case [6]), we can obtain the square root of the experimental plot $V_p^{(n)}(H_{max})$, and then determine the entire spectrum of the generated frequencies and the voltages corresponding to them by means of (2). The values of $V_p^{(n)}$ obtained in this fashion agree well with experiment (accurate to ~ 10%). A detailed description of the results of the experimental investigation will be published later.

Thus, unlike the Pb-I-Pb tunnel structures described in [6], a strong interaction between the alternating Josephson current and the resonant mode of the strip resonator formed by the tunnel junction occurs in our tunnel structures. This is indeed the mechanism that causes such an effective coupling between the alternating Josephson current and the electromagnetic field, and this in turn has made it possible to observe directly the photon emission in [3].

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# TEMPERATURE DEPENDENCE OF HYPERFINE INTERACTION LINES IN EPR SPECTRA OF PHOSPHORUS IN SILICON

B. G. Zhurkin and N. A. Penin
P. N. Lebedev Physics Institute, USSR Academy of Sciences
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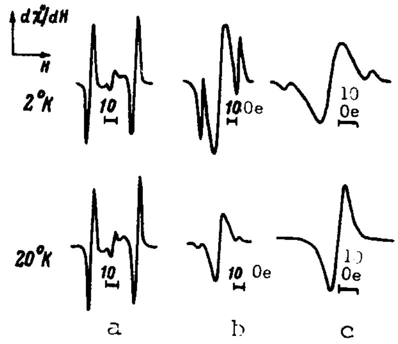
In an investigation of the temperature dependence of the spectra of electron paramagnetic resonance (EPR) in n-type silicon doped with phosphorus, it was observed that the hyperfine-interaction lines behave differently in spectra of samples with different phosphorus concentrations. The figure shows the EPR spectra dX"/dH = f(H) at two temperatures (2 and 20°K) for samples with phosphorus donor impurity concentration $N_D = 2 \times 10^{17}$, $4.5 \times 10^{17}$, and $6 \times 10^{17}$ cm$^{-3}$. We see that, unlike the sample with $N_D = 2 \times 10^{17}$ cm$^{-3}$, in the samples with $N_D = 4.5 \times 10^{17}$ and $6 \times 10^{17}$ cm$^{-3}$ the intensity of the hyperfine-interaction lines decreases rapidly with increasing temperature.

The difference in the temperature dependence of the hyperfine interaction lines in the investigated silicon samples is connected, in our opinion, with the different nature of the paramagnetic centers which make the main contribution to these lines at different phosphorus concenterations.



EPR spectra: a - $N_D = 2 \times 10^{17}$, b - $N_D = 4.5 \times 10^{17}$, c - $N_D = 6 \times 10^{17}$ cm$^{-3}$.

At a donor concentration $N_D = 2 \times 10^{17}$ cm$^{-3}$, the hyperfine interaction lines are due principally to isolated atoms of phosphorus, since the overlap of the wave functions is insignificant at this concentration [1]. There is therefore no exchange of electrons between the neighboring atoms to lead to a decrease in the number of bound electrons, and consequently the intensity of the hyperfine interaction lines is practically independent of the temperature.

At donor concentrations $N_D = 4.5 \times 10^{17}$ and $6 \times 10^{17}$ cm$^{-3}$, there is considerable overlapping of the wave functions, as evidenced by the presence of an intense central line in the spectrum. In this case the hyperfine interaction lines are due essentially to groups of interacting atoms [1]. As a result, an increase in the temperature causes an increase in the frequency of the jump between atoms, i.e., an intensification of the delocalization of the electrons. As a result, an increase in temperature is accompanied by a decrease in the intensity of the hyperfine interaction lines and an increase in the intensity of the central line.