increasing the pump power. Using the fact that one of the pump bands (841 nm) is close to the emission band of gallium arsenide, we have tested a system in which the pumping was produced by such a semiconductor emitter. No appreciable improvement was obtained, however, first, because the radiation power was insufficient (~ 2 mW) and, second, because the 841 nm band corresponds to the transition $^4I_{13/2} \rightarrow {}^4S_{3/2}$, which is strongly L-forbidden ($\Delta L = 6$) and J-forbidden ($\Delta J = 10$), and is therefore of relatively low efficiency.

We have also observed in the $CaWO_4$ activated with Er^{3+} another scheme of stepwise transitions, differing from that of Fig. 1 in that the system goes over from the ground state, with absorption of ~ 980 nm radiation, into the $^4I_{11/2}$ state, the higher excited states remaining the same in this scheme.

The effect of stepwise excitation of fluorescence was obtained by us also in single crystals of $PbMoO_4$ activated with Er^{3+}, at a concentration 0.5%. The effect was observed at the same transitions and wavelengths as in $CaWO_4:Er^{3+}$.

[1] M. R. Brown and W. A. Shand, Phys. Rev. Lett. 12, 367 (1964).

[2] N. Bloembergen, Phys. Rev. Lett. 2, 84 (1959).

[3] A. M. Prokhorov, JETP 40, 1384 (1961), Soviet Phys. JETP 13, 973 (1961).

NEW ACOUSTIC RESONANCE IN AN OBLIQUE MAGNETIC FIELD

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References [1,2] report the observation of resonant oscillations of the absorption coefficient of sound in tin; these oscillations are connected with the drift of the electrons along the wave vector $\mathbf{k}$. The experiments were carried out with orthogonal $\mathbf{k}$ and $\mathbf{H}$ [$\mathbf{H}(0, 0, H_z)$ is the magnetic field vector], with the drift of the carriers along $\mathbf{k}$ being ensured by the open Fermi surface. Later [3,4] an analogous resonance was investigated in cadmium and in zinc, where the Fermi surfaces likewise have open directions.

In the case of closed surfaces, the resonance connected with the carrier drift can be obtained on the non-extremal orbits of the electrons in a magnetic field which is inclined to $\mathbf{k}$. Theoretically such an effect was considered in [5,6]. The resonance condition consists in the fact that when $\ell \gg \lambda$ the electron displacement during the time of one revolution in the magnetic field is equal to

$$s = n\lambda \qquad (1)$$

where n = 1, 2, 3, ..., λ - wavelength of sound, and ℓ - electron mean free path.

The oscillations are produced on the limiting trajectories (Fig. 1). The contribution to the absorption of sound is made by the electrons for which $\mathbf{k}\cdot\mathbf{v} = 0$ ($\mathbf{v}$ is the electron velocity vector), and consequently the only orbits that can participate in the resonance are those having common points with the line $\mathbf{k}\cdot\mathbf{v} = 0$ on the Fermi surface. The state density of carriers with a given displacement along $\underset{\sim}{k}$ has an extremum near the contact between the lines $\underset{\sim}{k}\cdot\underset{\sim}{v} = 0$ and $p_z = p_{z.lim}$, and consequently these are precisely the orbits which determine the resonant frequency.

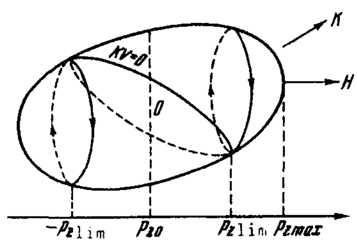

Fig. 1. Schematic diagram of the limiting trajectories and of the line $\underset{\sim}{k}\cdot\underset{\sim}{v} = 0$ on a closed convex surface $\epsilon(p) = \epsilon_f$

To observe the effect we set up experiments involving the measurement of the coefficient of sound absorption in single-crystal antimony. The samples for the measurements were grown from brand V-000 metal in the form of discs of thickness $\alpha \simeq 1$ mm with the normal along the binary axis. At the temperature of liquid helium, the ratio l/λ, estimated from the number of oscillations of the geometrical resonance [7], reached 150 (at $\omega/2\pi = 5 \times 10^8$ cps). The binary axis of the crystal coincides with the direction of the wave vector $\underset{\sim}{k}$. The experiment was set up in such a way that the magnetic field vector could be rotated in the plane of the binary and trigonal axis.

Figure 2 shows records of the derivative of the absorption coefficient $d\Gamma/dH$ for one of the orientations of $\underset{\sim}{H}$, as a function of the reciprocal magnetic field. The curves A and B were recorded at sound frequencies 5×10^8 cps and 3×10^8 cps, respectively. In weak fields the lines have a sinusoidal form. With increasing field intensity, they narrowed down, approaching a somewhat asymmetrical Lorentz shape. The oscillations have the same period in the reciprocal magnetic field. The change in the frequency of sound leads to a shift of the corresponding resonance lines in proportion to the frequency. The amplitude of the oscillations in weak fields increases with increasing magnetic field intensity, and then begins to drop. When the orientation of $\underset{\sim}{H}$ is changed, a well pronounced anisotropy of the period is observed, and for some field directions new oscillating components appear producing the characteristic picture of beats.

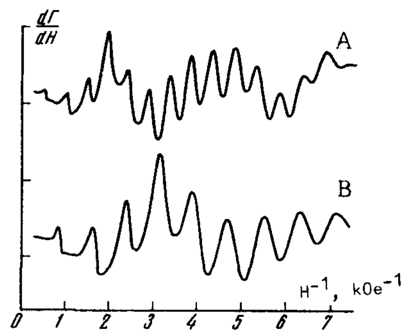

Fig. 2. Plot of the derivative of the longitudinal-sound absorption coefficient d /dH vs. the reciprocal magnetic field. The angle between $\underset{\sim}{k}$ and $\underset{\sim}{H}$ is 25°, $\underset{\sim}{k}$ is along the binary axis, $\underset{\sim}{H}$ is in the plane of the binary and triagonal axes.
A - $\omega/2\pi = 5 \times 10^8$ cps;
B - $\omega/2\pi = 3 \times 10^8$ cps.

The form and the amplitude of the derivative $d\Gamma/dH$ near resonance is described, according to [5], by the formula

$$\frac{d\Gamma}{dH} \sim \frac{\Gamma_0(kR)^{1/3}}{\gamma H} \frac{\gamma^2}{\gamma^2 + \Delta^2} \tag{2}$$

where $\gamma = 1/\Omega\tau$, Ω - cyclotron frequency, $\tau = 1/\nu$ - relaxation time, $\Delta = \delta H/H$ - relative detuning, $k = |\underset{\sim}{k}|$, R - cyclotron radius, Γ_0 - absorption in the absence of a magnetic field. At resonance and when $\gamma < 1$ the amplitude of the derivative decreases with increasing field like $H^{-1/3}$, in qualitative agreement with the results of our measurements.

The period of the oscillations determines the product of the average electron velocity $\bar{v}$ along the direction of the field $\underset{\sim}{H}$ by the cyclotron mass m_c:

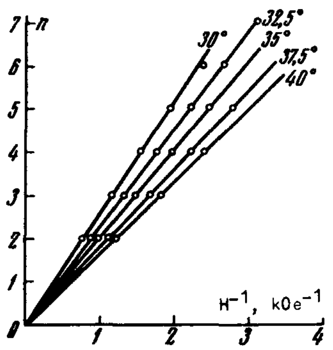

Fig. 3. Dependence of the number of oscillations $\underline{n}$ on the reciprocal magnetic field for several different directions of the vector $\underset{\sim}{H}$ in the plane of the binary and trigonal axes of the crystal. $\underset{\sim}{k}$ is parallel to the binary axis. The numbers on the lines indicate the angle between $\underset{\sim}{k}$ and $\underset{\sim}{H}$.

$$m_c \bar{v} = \frac{1}{2\pi} \frac{\partial S(p_z)}{\partial p_{z.\mathrm{lim}}} = \frac{e}{ck_z \Delta H^{-1}} \qquad (3)$$

The initial phase of the oscillations, in accordance with condition (1), should be equal to zero. Figure 3 shows the numbers of the oscillations to be functions of the reciprocal magnetic field for several directions of the vector $\underset{\sim}{H}$. It is seen from the figure that the initial phase of all series of oscillations is indeed equal to zero.

In conclusion we are sincerely grateful to E. A. Kaner for useful discussions and also to N. D. Belousov for help with the measurements.

[1] Galkin, Kaner, and Korolyuk, DAN SSSR 134, 74 (1960), Soviet Phys. Doklady 5, 1002 (1961).

[2] Galkin, Kaner, and Korolyuk, JETP 39, 1517 (1960), Soviet Phys. JETP 12, 1055 (1961).

[3] J. D. Gavenda and B. C. Deaton, Phys. Rev. Lett. 8, 208 (1962).

[4] B. C. Deaton, Phys. Rev. 16, A1096 (1964).

[5] Kaner, Peschanskii, and Privorotskii, JETP 40, 214 (1961), Soviet Phys. JETP 13, 147 (1961).

[6] E. A. Kaner, JETP 43, 216 (1962), Soviet Phys. JETP 16, 154 (1963).

[7] A. A. Galkin and A. P. Korolyuk, JETP 38, 1688 (1960), Soviet Phys. JETP 11, 1218 (1960).

CASIMIR OPERATORS FOR THE ORTHOGONAL AND SYMPLECTIC GROUPS

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As is well known [1], the name orthogonal group O(n) is given to the group of linear transformations which conserve the quadratic form $\Sigma_{i=1}^{n} (x^i)^2$; analogously, the symplectic group Sp(2n) consists of unitary transformations which conserve the bilinear form $\Sigma_{i=1}^{n} (y^i x^{-i} - y^{-i} x^i)$. The simplest orthogonal groups O(2), O(3), and O(4) have numerous applications in physics; the orthogonal groups of higher order, as well as the symplectic groups, are used in the classification of states in the nuclear-shell model [2]. This frequently raises the problem of finding invariant operators (the so-called Casimir operators) which can be constructed from the generators of the given group. The most important in physics is the quadratic Casimir operator C_2, the eigenvalues of which were obtained by Racah [2]. Explicit expressions for the eigenvalues of the operators C_p with $p > 2$ were never published (with the exception of the operator C_4 for the group Sp(4), see [3]). We present below a solution of this problem in general form.