form a complete set of independent invariant operators. A special situation arises for the grou O(2n): in order for the eigenvalues of the invariant operators to characterize the irreducible representation uniquely, the operator C_{2n} must be replaced by the operator C'_n:

$$C'_n = \epsilon^{j_1 j_2 \cdots j_n}_{i_1 i_2 \cdots i_n} X^{i_1}_{j_1} X^{i_2}_{j_2} \cdots X^{i_n}_{j_n} \tag{7}$$

which is analogous to the pseudoscalar $\epsilon_{\mu\nu} u_{\mu\nu} u_{\rho\sigma}$ in the Lorentz group. The eigenvalues of C'_n are:

$$C'_n(f_1, \ldots, f_n) = (-1)^{\frac{n(n-1)}{2}} 2^n n! \ell_1 \ell_2 \ldots \ell_n \tag{8}$$

In conclusion we note that not all representations of the groups O(2n) and O(2n + 1) can be described by a Young tableau (the orthogonal group includes spinor representations). However, all the preceding formulas are valid in this case, too, if f_i is taken to mean the eigenvalue of the diagonal operator X^i_i for the highest-order vector of the irreducible representation.

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LASER WITH RADIATION DIAGRAM OF DIFFRACTION WIDTH

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As is well known [1,2], a large number of different modes whose resonant frequencies lie inside the luminescence line of the active medium, are excited simultaneously in a laser. This not only affects adversely the coherence of the radiation, but also distorts appreciably the directivity pattern, which becomes broad and jagged. The latter circumstance makes it difficult to use lasers for many scientific and technical applications.

The existing methods of selecting the oscillations for the purpose of inducing lasing conditions in one of the lower modes (TEM_{00q}) are based on the insertion of various optical elements into the resonator, and are incovenient in that they cause large losses. In this connection we consider a mode selection method based on choosing a resonator configuration such that the diffraction losses of the proper modes are essentially different. This property is possessed, in principle, by an ordinary confocal resonator, but only if its dimensions correspond to very small Fresnel numbers $N < 1$, where $N = r^2 L\lambda$, r is the radius of the mirrors, and L is the length of the resonator. For $r \sim 1$ cm, $\lambda = 10^{-4}$ cm, and $N \sim 1$ the resonator length L should be more than 100 meters, which cannot be realized in the case of gas lasers.

The resonator proposed below consists of a set of plane and spherical mirrors separated by a distance L which is close to the radius of curvature R of the spherical mirror. On the basis of the generalized theory of resonators [3] it can be shown that such a configuration, with $R > L$, is equivalent to a confocal system with mirrors having a curvature radius $R_e = [4L(R - L)]^{1/2}$. The diffraction losses of the resonator in question were determined by the equivalent Fresnel number

$$N_e = \frac{r^2}{\lambda L}\sqrt{\frac{L}{R}\frac{R - L}{R}}$$

In this case the Fresnel number can be made quite small by suitable choice of the distance between mirrors, and the geometrical dimensions of the resonator still remain acceptable.

Thus, with a resonator of length sufficiently close to the radius of curvature of the spherical mirror, the Q of the fundamental mode of the oscillations will be essentially higher than the Q of the other modes (see the dashed curves on Fig. 1), making it possible to realize single-mode lasing.

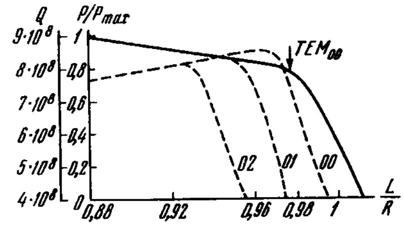

Fig. 1

This deduction is confirmed by the results of experiments carried out with a helium-neon laser. The laser had a discharge tube 1100 mm long and 4 mm in diameter, and, depending on the mirrors employed, could emit at either 0.63 or 1.15 μ. The radius of the curvature of the spherical mirror was 1300 mm.

We investigated the dependence of the output power of the generator and the number of different modes excited in it on the resonator wavelength. The radiation power was measured with a calorimeter, and the field structure of the laser emission was examined simultaneously either visually or with the aid of an electron-optical converter. The emission spectrum was investigated by separating the difference frequencies between different modes. The difference frequencies were produced at the output of an FEU-62 photomultiplier on whose cathode the laser emission was focused. They were located in the radio band and were investigated with type S4-9 and S4-8 spectrum analyzers.

As seen from Fig. 1, the output power of the generator decreased insignificantly when the resonator length was increased to values close to the radius of curvature of the spherical mirror. However, the pattern of the radiation field and the spectrum of the difference frequencies experienced appreciable changes. At distances considerably smaller than the radius of curvature, many different transverse modes were observed, and accordingly a large number of lines in the difference-frequency spectrum. With increasing L, the number of modes gradually decreased. Finally, only one fundamental transverse mode, TEM_{00}, remained at $L/R \geq 0.975$. The spectrum then contained only the frequency near 120 Mcs, corresponding to beats between the longitudinal modes for one lower transverse mode. The instant of appearance of the single mode is marked in Fig. 1 by the arrow. The laser power amounted in that case to 0.7 - 0.8 of the maximum value, and reached 4 MW at 1.15 μ and 2.5 MW at 0.63 μ. The change in the character of generation with increasing L/R is confirmed by the calculation of the values of Q of the lowest modes. We see that the value of $L/R = 0.975$, at which the single-mode generation begins, corresponds to the instant when the Q for the fundamental mode TEM_{00} begins to exceed appreciably the values of the Q of the higher modes. Further change in radiation power corresponds to the variation of the fundamental-mode Q curve.

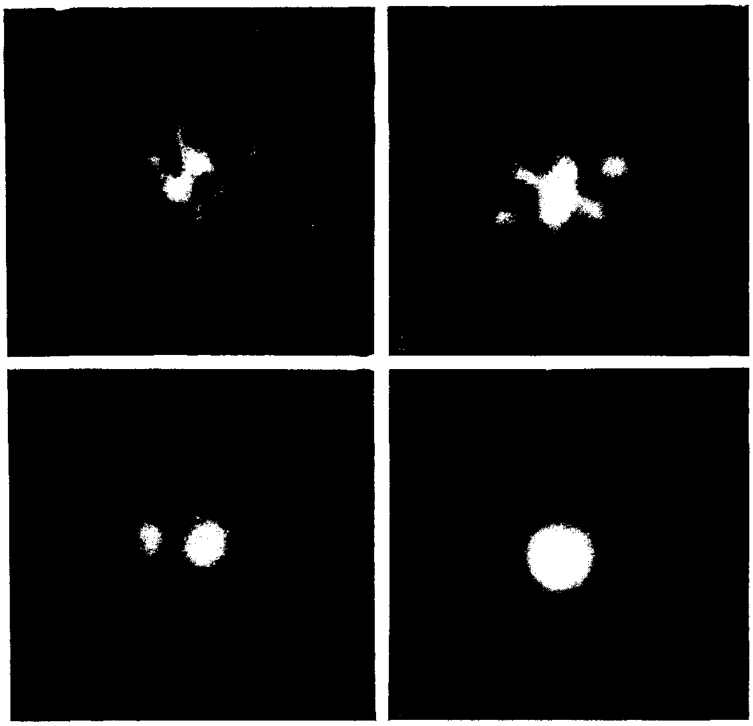

Fig. 2

Single mode generation makes it possible to realize a narrow directivity pattern, the width of which corresponds to the diffraction limit. We have investigated the directivity pattern on the plane-mirror side, measuring the distribution of the field in the focal plane of the objective which focused the laser radiation. The picture obtained in the focal plane was photographed, after which the resultant film was measured photometrically. The photographs corresponding to different values of resonator lengths (L/R = 0.880, 0.958, 0.968, and 0.975) are shown in Fig. 2. Figure 3 shows the directivity pattern for the multiple-mode operation and two cases of fundamental-mode generation. We see that at $L/R = 0.880$ (curve 1), when several modes are generated simultaneously, the directivity pattern is ragged. When $L/R = 0.975$ (curve 2), single-mode operation sets in (see also Fig. 1), and the width of the diagram at the half-power level narrows down to 4.3', in good agreement with calculation. Indeed, using the well-known formula [2]

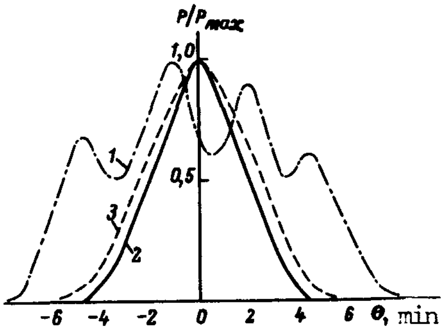

Fig. 3

$$\theta_{0.5} = 0.94\sqrt{(\lambda/R_e)}$$

we obtain for this case $\theta_{0.5} = 4.1'$.

With increasing resonator length, the directivity remains smooth, but broadens. For $L/R = 0.987$ (Fig. 3, curve 3), its width amounts to 5.2'. The corresponding calculated value is 4.9'.

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