RELATIVISTIC GENERALIZATION OF SU(3) SYMMETRY. BARYON CURRENT.

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SU(3) symmetry of elementary particles has a natural explanation within the framework of the composite model, based on a triplet of quarks [1]. Inasmuch as the quark has spin 1/2, its wave function should be a bispinor of the Lorentz group $\psi_{a\alpha}$ (a - index of the triplet, α - index of the bispinor). However, if we construct particles from such quarks, then no connection whatever is obtained between the unitary and the spin properties. The connection arises only in a theory which treats all the 12 components ψ_A as equivalent.

We can choose for the baryon wave function a symmetrical spinor of third rank $\psi_{A,B,C}$ (A,B,C = 1, ..., 12), which has 364 components. When the moderately-strong interaction is turned on, this supermultiplet breaks up into SU(3) multiplets with definite values of spin and parity [2]

$$364 = \underline{10}(3/2^+) + \underline{8}(1/2^+) + \underline{1}(1/2^-) + \underline{8}(3/2^-) \tag{1}$$

Such a supermultiplet composition ties in very well with the experimental data [3] and justifies our choice of the baryon wave function. Its explicit form is

$$\psi_{ABC} = \sqrt{(2/3)}(e_{abd}\mathcal{P}_{\alpha\beta}B^d_{c,\gamma} + e_{bcd}\mathcal{P}_{\beta\gamma}B^d_{a,\alpha} + e_{cad}\mathcal{P}_{\gamma d}B^d_{b,\beta}) \tag{2}$$

Here e_{abd} is a completely antisymmetrical tensor, B^a_b is the bispinor of the baryon octet, and

$$\mathcal{P}_{\alpha\beta} = \left[\frac{-i\hat{p} + m}{2m}\gamma_5 C\right]_{\alpha\beta} \tag{3}$$

is an antisymmetrical spinor of second rank, C - charge conjugation matrix, and p and m - momentum and mass of the baryon.

It is convenient to expand the baryon current in terms of the system of 16 Dirac matrices

$$J^A_B = \frac{1}{2}\delta^A_B\bar{\psi}^{CDE}\psi_{CDE} - \bar{\psi}^{ADE}\psi_{BDE} = S^a_b\delta_{\beta\alpha} + (v_\mu)^a_b(\gamma_\mu)_{\beta\alpha} + (T_{\mu\nu})^a_b(\gamma_{\mu\nu})_{\beta\alpha} + $$
$$+ (A_\mu)^a_b(i\gamma_5\gamma_\mu)_{\beta\alpha} + P^a_b(\gamma_5)_{\beta\alpha} \tag{4}$$

with coefficients

$$(J_i)^a_b = \frac{-P^2}{6m^2}\bar{u}(p_2)\Gamma_i u(p_1)(D + F)^a_b + \frac{\omega_i}{12m^2}(\bar{u}_2\hat{P}\Gamma_i\hat{P}u_1)D^a_b -$$
$$- \frac{(\bar{u}_2 u_1)}{6}\mathrm{Sp}\,\mathcal{P}\bar{\mathcal{P}}\Gamma_i(D - F)^a_b \tag{5}$$

in which F^a_b and D^a_b are known combinations of wave functions (see, for example, [4]) $P = p_1 + p_2$, $\Gamma_i = 1, \gamma_\mu, \gamma_{\mu\nu}, i\gamma_5\gamma_\mu, \gamma_5$, $\bar{\mathcal{P}}_{\alpha\beta}$ - adjoint spinor, $\omega_i = +1$ for S, V, P and - 1 for T and A.

We note first of all that in the static limit the obtained expression for J^A_B reduces to two currents of the Shekhter type [4] with different spatial parity. Thus, SU(6) symmetry is contained in the given theory as a limiting case.

The electromagnetic current of the baryon octet is

$$J_\mu^{(em)} = \langle\gamma_\mu\rangle(F_b^a + \frac{3q^2}{4m^2}D_b^a) + \frac{1}{2m}\langle\gamma_{\mu\nu}q_\nu\rangle(3D_b^a + F_b^a + \frac{q^2}{2m^2}F_b^a) \qquad (6)$$

and in the static limit ($q^2 \to 0$) is of the pure F-type in agreement with the hypothesis on the octet character of the electromagnetic current [5], making it possible to calculate the magnetic moments of the baryons

$$\mu = (3D + F)\mu_{nuc} \qquad (7)$$

Formula (7) contains not only the moment ratios known from SU(6) symmetry, but also their absolute values

$$\mu_p = +3\mu_{nuc}, \qquad \mu_n = -2\mu_{nuc} \qquad (8)$$

which agree within 5 - 10% with experiment.

When $q^2 \neq 0$, formula (6) leads to the following results:

a) Equality of the form factors of the proton $F_1^{(p)}(q^2) = F_2^{(p)}(q^2)$.

b) Vanishing of the form factor of the neutron, $G_E^{(n)}(q^2) = 0$.

c) The value of the derivative

$$dG_1^{(n)}(q^2)/dq^2|_{q^2=0} = \frac{1}{2m^2} = 0.022\ F^2 \qquad (9)$$

agrees well with experiment [6] (within ± 5%). From (9) we can obtain the depth of the neutron-electron interaction well $U_{ne} = -4{,}270$ eV [7].

We can consider weak current in the same manner.

After this work was completed, we learned of the paper by Salam et al. [8], in which the same problems are considered from a different point of view. Their derivations differ essentially from ours in many respects.

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