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A POSSIBLE GENERALIZATION OF EINSTEIN'S EQUATIONS

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The principle whereby physical media are described with the aid of a certain tensor and equations that relate the tensor with a metric, a principle realizable for media with energy-momentum tensors of definite structure in general relativity, will be extended in this article, within the framework of general relativity, to all media that differ from zero-curvature space-time and are characterized by tensor invariants.

For this purpose, we introduce a tensor with the following properties:

(i) It is a rational function of a metric tensor and its first and second derivatives linear in the second derivatives.

(ii) It satisfies the conservation law (the vanishing of the covariant divergence by virtue of relations between this tensor and the metric tensor).

(iii) It vanishes if and only if space-time has no curvature.

The meaning of these requirements is obvious. The first two of these were used by Einstein to find the tensor G_{jk}.

Apart from trivial transformations involving multiplication by a number and symmetrization, requirements (i) - (iii) define the following tensor:

$$\left.\begin{array}{c} G_{jk\ell m} = - R_{jk\ell m} + W_{jk\ell m} - (1/2)g_{jk\ell m}G, \\ W_{jk\ell m} \overset{\text{def}}{=} g_{jm}G_{k\ell} + g_{k\ell}G_{jm} - g_{j\ell}G_{km} - g_{km}G_{j\ell}, \\ g_{jk\ell m} \overset{\text{def}}{=} g_{jm}g_{k\ell} - g_{j\ell}g_{km}, \quad G_{jk} \overset{\text{def}}{=} R_{jk} - (1/2)g_{jk}R, \quad R_{jk} \overset{\text{def}}{=} R^{\ell}_{jk\ell}, \quad G \overset{\text{def}}{=} G^{\ell}_{\ell} \end{array}\right\} \quad (1)$$

where g_{jk} and $R_{jk\ell m}$ are metric and Riemann tensors. In view of

$$G_{jk\ell m} = - G_{kj\ell m} = - G_{jkm\ell} = G_{\ell mjk}, \quad G_{jk\ell m} + G_{j\ell mk} + G_{jmk\ell} = 0 \quad (2)$$

only 20 components of the tensor $G_{jk\ell m}$ are independent, and from

$$G^{j}_{k\ell m|j} = 0 \quad (3)$$

follows the vanishing of the covariant divergence with respect to k, ℓ, and m, too.

The tensor $G_{jk\ell m}$ turns out to have the remarkable property (not violated by symmetrization):

$$G^{\ell}_{jk\ell} = G^{\ell}_{j\ell k} = G_{jk} \quad (4)$$

i.e., its contraction gives the Einstein tensor.

We propose that for an arbitrary medium the tensor $G_{jk\ell m}$ should play the same role as the tensor G_{jk} in Einstein's equations:

$$G_{jk} = -\kappa T_{jk} \tag{5}$$

This means that the macroscopic description of an arbitrary medium should be realized by a fourth-rank tensor $T_{jk\ell m}$ having the symmetry properties (2) and defining a metric by virtue of the relations

$$G_{jk\ell m} = -\kappa T_{jk\ell m} \tag{6}$$

In view of (4), their contraction gives Einstein's equations (5). Consequently, the energy-momentum tensor is a contraction of the tensor $T_{jk\ell m}$. Since (6) are differential equations with respect to the components of a metric tensor, the latter depends not on the local properties of the medium (described by $T_{jk\ell m}$), but by their distribution in space-time. Therefore the tensor $T_{jk\ell m}$, like the tensor T_{jk} in Einstein's equations, expresses not the local properties of the metric, but characterizes objects that differ from it.

In view of (3) and (5), there follow from (6) the relations:

$$T^{j}_{k\ell m|j} = 0 \tag{7}$$

which determine the distribution of the invariants of the tensor $T_{jk\ell m}$ in space-time, i.e., the equations of motion of the medium. Only 20 of these are independent, since the tensor $T_{jk\ell m}$ satisfies identities of the type (2).

Let us discuss some individual cases.

Media whose volume elements have a rest mass are characterized by the existence of a unique local co-moving reference frame at each point in the medium. This takes place if and only if $T_{jk} \overset{\text{def}}{=} T^{\ell}_{jk\ell}$ is a tensor whose time-like eigenvalue differs from the space-like ones. Media satisfying this condition can be called ordinary matter. The physical reference frame is always understood to consist of bodies made up of ordinary matter. Therefore the existence and uniqueness of the co-moving reference frame signifies that motion of ordinary matter relative to ordinary matter is uniquely defined.

The condition $T_{jk} \neq 0$, $T^{\ell}_{\ell} = 0$ is satisfied by an electromagnetic field.

Finally, when $T_{jk} = \mu g_{jk}$, where μ is a constant, the medium has macroscopically the properties of vacuum [4]: any local reference frame is co-moving for it, so that any interaction between this medium and ordinary matter does not depend on the velocity of the latter (the relativity principle). We shall call this medium μ-vacuum. When $\mu = 0$ it constitutes ordinary vacuum. The space-time corresponding to it is the Einstein space in the sense of Petrov [1].

The three indicated media, which differ in the properties of the contractions of the tensor $T_{jk\ell m}$, belong to the three main types of media predicted by general relativity. Let us consider a world of ordinary matter and an electromagnetic field in a μ-vacuum. Within the framework of the consequences of the system (6), there is a unique possibility of describing gravitational interactions, viz., they result from transport, defined by the equations of motion (7), of the invariants of the tensor $T_{jk\ell m}$ of μ-vacuum. It is easy to show that the vacuum invariants which change in space-time cannot be interpreted as being energy-momentum components, since they do not influence the structure of the energy-momentum tensor of μ-vacuum (for example, when $\mu = 0$ we also have $T_{jk} = 0$). From this point of view, such problems as the determination of the density of the energy of a gravitational wave cannot have a solution. We can speak only of loss or acquisition of energy by a system, made up of ordinary matter and an electromagnetic field, interacting with the μ-vacuum. In the latter, on the other hand, the conservation and propagation laws have a form different than for matter with $T_{jk} \neq \mu g_{jk}$. The vacuum invariants were studied by Petrov [1]. In the particular case $\mu = 0$, their transport was investigated by Pirani [2] and later by Ehlers and Sachs [3].

Inasmuch as Einstein's equations (5) follow from a system (6), the latter lead also to the results which follow from Einstein's equations. The system (6), however, describes directly also processes in media which have the properties of vacuum, as well as in mixed media in which a vacuum component is essential, whereas such processes can be described with the aid of (5) at best only indirectly (for example, as a consequence of specifying the Cauchy data on some space-like surface). Since, however, the system (6) conserves furthermore the unity of geometry and physics, as is characteristic of general relativity, it seems to us that it can be regarded as a natural possible generalization of Einstein's equations.

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NEODYMIUM-GLASS LASER WITH PULSED Q SWITCHING

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We used an electron-optical shutter to modulate the Q of a neodymium-glass laser. This shutter ensured a shorter Q-switching time than the previously used device with rotating prism [1]. The components of the laser were: a mirror with reflection coefficient 98% at wavelength 1.06 μ, the shutter, and two neodymium-glass (KGSS-7) rods each 120 mm long and 10 mm in diameter, with parallel end surfaces. The excitation was by means of two helical lamps with pump energy 8 kJ each and duration 600 μsec (at the 0.3 level). The shutter consisted of two crossed polarized prisms and a Kerr cell, which was controlled by a pulse with a lifetime of 5 nsec and a duration 600 nsec. The pulse was generated with a long-line generator [2].