THE QUARK HYPOTHESIS AND RELATIONS BETWEEN CROSS SECTIONS AT HIGH ENERGIES

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Numerous attempts are now under way to apply higher symmetries to the process of particle interaction. In spite of some success [1], the physical meaning of the operations performed remains unclear. We propose here a physically lucid approach to the account of symmetry in particle interaction at high energies. It is assumed that the particles are made up of quarks [2] and that the wave function of the free particles transforms in accordance with SU(6) symmetry [3] in the particle rest frame. It is assumed that fast-particle collision process consists principally of single scattering of a quark of one particle by a quark of the other.

The validity of such an approximation can be explained by means of the following model. Let us assume that the heavy quarks are on the bottom of a deep vector well of rectangular form [4]. In this case we can assume that the quark wave function is equal to zero outside the well and that the quarks inside the well are nonrelativistic if the well is sufficiently broad. If we assume that the wells do not change appreciably during the course of the collision of the relativistic particles and that the interaction radius of the bound quark with a quark (antiquark) decreases with increasing energy, then the main contribution is made by single scattering of the quark (antiquark) of one particle by the quark of the other, provided the momentum of the colliding particles is much larger than the momentum of the quark inside the well. According to estimates obtained in this model, the interaction radius of a bound quark with a quark (antiquark), determined from the total cross section, is smaller by a factor 2 - 3 than the interaction radius of the nucleons at high energies, thus favoring this approximation. We note that the physical meaning of our approximation is entirely different from the case of the deuteron, since the quarks collide inside the well. The amplitude for relativistic quark-quark (antiquark) scattering in a broad rectangular well can be written in the c.m.s. as follows:

$$\begin{aligned} M_{ij} = a + b(\sigma_i, \sigma_j) + c(\sigma_i + \sigma_j, \nu) + d(\sigma_i K)(\sigma_j K) + \\ + e(\sigma_i N)(\sigma_j N) + (F_i F_j)[a_F + b_F(\sigma_i \sigma_j) + c_F(\sigma_i + \sigma_j, \nu) + \\ + d_F(\sigma_i K)(\sigma_j K) + e_F(\sigma_i N)(\sigma_j N)] \end{aligned} \quad (1)$$

where σ are Pauli matrices, $\nu = [n \times n']/|n \times n'|$, $\underset{\sim}{N} = (n - n')/|n - n'|$, $\underset{\sim}{K} = (n + n')/|n + n'|$, $\underset{\sim}{n}$ and $\underset{\sim}{n}'$ are unit vectors in the directions of the quark momenta before and after scattering in the c.m.s., and F_i and F_j are Gell-Mann's 8 operators for the quarks.

Knowing the form of the "quark" amplitude, we can express the particle scattering amplitude in terms of M_{ij}; averaging over the coordinate wave function of the quark inside the particle is easy if the momentum of the colliding particles is much larger than the momentum of the quark inside the well. Assuming that the free nucleons belong to the representation $\underline{56}$ and the mesons to representation of the 35 group of SU(6), we can express all the amplitudes in terms of M_{ij}.

Using the optical theorem, we obtain the following relations between the total cross sections:

$$\sigma_{pn} - \sigma_{p\Sigma^+} = \sigma_{p\Sigma^-} - \sigma_{p\Xi^0} = 0 \tag{2}$$

$$\sigma_{p\lambda} = 2\sigma_{pn} - \sigma_{pp} = (1/2)(\sigma_{p\Sigma^+} + \sigma_{p\Sigma^-}) = (1/3)(2\sigma_{p\Xi^-} + \sigma_{pp}) \tag{3}$$

$$(1/2)(\sigma_{K^+p} - \sigma_{K^-p}) = \sigma_{K^0p} - \sigma_{\bar{K}^0p} = \sigma_{\pi^+p} - \sigma_{\pi^-p} \tag{4}$$

$$\sigma_{K^+p} + \sigma_{K^-p} = (1/2)(\sigma_{K_0p} + \sigma_{\bar{K}_0p} + \sigma_{\pi^-p} + \sigma_{\pi^+p}) \tag{5}$$

These relations can be obtained from the pure SU(3) symmetry, confining ourselves in the t-channel to the representations $\underline{1}$ and $\underline{8}_F$ for the baryon-antibaryon vertex and to $\underline{1}$, $\underline{8}_F$, and 8_D for the meson-meson vertex. Relations (2 - 3) cannot be verified experimentally, and (4) is the Jonson-Treiman relation [1]. The left side of (5) is equal to 40 mb at a momentum P = 18 BeV/c, while the right side is equal to 44 mb. If we assume that the "quark" amplitudes are the same for meson-nucleon and nucleon-nucleon interactions, then we can assume the additional relations

$$\sigma_{pp} - \sigma_{pn} = \sigma_{K^-p} - \sigma_{K^-n} \tag{6}$$

$$\sigma_{pp} - \sigma_{pn} = (1/2)[\sigma_{\pi^+p} - \sigma_{K^-n}] \tag{7}$$

$$\sigma_{pp} - \sigma_{pn} = \sigma_{\pi^-p} - \sigma_{K^-p} \tag{8}$$

$$\sigma_{p\bar{p}} - \sigma_{p\bar{n}} + \sigma_{pp} - \sigma_{pn} = \sigma_{\pi^-p} - \sigma_{\pi^+p} \tag{9}$$

$$2(\sigma_{pp} + \sigma_{p\bar{n}}) - (\sigma_{pn} + \sigma_{p\bar{p}}) = 3\sigma_{\pi^+p} \tag{10}$$

$$2\sigma_{pn} - \sigma_{pp} + 5\sigma_{p\bar{n}} - 4\sigma_{p\bar{p}} = 3\sigma_{K^+n} \tag{11}$$

To judge whether relations (6 - 11) are satisfied, it is reasonable to compare them with experiment at maximal energies, when the cross sections are almost constant. This comparison was made in accordance with the data of [5]. The left sides of (6 - 8) are equal to 0.6 mb at P = 18 BeV/c. The right side of (6) is equal to 1.3 mb, whereas that of (7) is approximately zero, and that of (8) is 4 mb. Relation (9) is satisfied in almost the same manner as (6). In the case of (10) at 18 BeV/c, the right and left sides are 72 and 88 mb, respectively. For (11) the corresponding values are 54 and 88 mb. We see thus that relations (10 - 11) are strongly violated. This can be attributed, for example, either to the breaking of the SU(3) symmetry or to the insufficient energy.

In addition, we can obtain relations for the cross sections averaged over the multiplet, which are conserved when SU(3) is broken:

$$\langle \sigma_{NN} \rangle + \langle \sigma_{N\bar{N}} \rangle = 3\langle \sigma_{\pi N} \rangle \tag{12}$$

$$[\langle \sigma_{NN} \rangle + \langle \sigma_{N\bar{N}} \rangle]\langle \sigma_{\pi\pi} \rangle = 2[\langle \sigma_{\pi N} \rangle]^2 \tag{13}$$

where $\langle \sigma \rangle$ is the cross section averaged over the multiplet. If we assume that in the limit of Pomeranchuk's theorem the cross section σ_{pp} tends to 39 mb while $\sigma_{\pi p}$ tends to 25 mb, then (12) is fairly well satisfied; an exact check is difficult, since $\langle \sigma_{NN} \rangle$ and $\langle \sigma_{N\bar{N}} \rangle$ are unknown. Equation (13) recalls the relation from reference [6]. In addition to relations (6 - 13) for the cross sections, we can obtain relations for the inelastic cross sections and for the cross sections for small-angle scattering with charge exchange:

$$(3/4)[\sigma_{\pi^- p \to \Delta^0 \pi^0} + (2/3)\sigma_{\pi^+ p \to \Delta^{++}\pi^0}] =$$

$$= \sigma_{K^- p \to \pi_0 \lambda} + \sigma_{\pi^+ n \to \lambda K^+} - (3/2)[\sigma_{K^- p \to \Sigma^+ \pi^-} + \sigma_{\pi^+ p \to K^+ \Sigma^+}] \tag{14}$$

$$\sigma_{K^- p \to \pi_0 \lambda} - (3/4)\sigma_{K^- p \to \Sigma^+ \pi^-} = (3/4)\sigma_{K^- p \to \Sigma^+_\delta \pi^-} \tag{15}$$

$$\sigma^{el}_{K^+ p} - \sigma^{el}_{K^- p} = [\sigma^{el}_{\pi^+ p} - \sigma^{el}_{\pi^- p} + \sigma^{el}_{K^0 p} - \sigma^{el}_{\bar{K}^0 p}] \tag{16}$$

An important feature of the proposed model is that the Gell-Mann operators enter the matrix elements in the first power, so that the transitions of p into Σ^-, Ξ, Σ^-_δ, and Ξ_δ are forbidden, in correspondence with octet dominance according to SU(3). In addition, the suppression of the production of particles of representation 10 in πN collisions can be related with the fact that this process is governed by the spin-flip amplitude. We note that these hindrances hold true also in the case when SU(3) is broken in the usual manner, and are in fair agreement with experiment [7]. If further experimental verification confirms the derived relations, this can be regarded as an argument in favor of the existence of quarks.

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[1] K. Jonson and S. B. Treiman, Phys. Rev. Lett. 14, 189 (1965); R. Good and Nguyen-huu Xuong, Phys. Rev. Lett. 14, 191 (1965).

[2] M. Gell-Mann, Phys. Lett. 8, 214 (1964); L. Zweig, CERN Preprint 8419/TH, 412, 1964.

[3] F. Gursey and L. Radicatti, Phys. Rev. Lett. 13, 173 (1964); A. Pais, Phys. Rev. Lett. 13, 175 (1964).

[4] Bogolyubov, Struminskii, and Tavkhelidze, JINR Preprint D-1968, 1965.

[5] S. L. Lindenbaum, Paper at 1964 Dubna High-energy Physics Conference.

[6] Gribov, Ioffe, Pomeranchuk, and Rudik, JETP 42, 1419 (1962), Soviet Phys. JETP 15, 984 (1962).

[7] H. Harari and H. Y. Lipkin, Phys. Rev. Lett. 13, 208 (1964); Meshkov, Snow, and Yodh, Phys. Rev. Lett. 13, 212 (1964).