DEPENDENCE OF THE SLOPE OF THE DIFFRACTION CURVES OF pp, $\bar{p}p$, K^+p, K^-p, π^+p, AND π^-p SCATTERING ON THE ENERGY

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It is shown that, starting with several GeV and continuing up to the highest energies attained in accelerators, the diffraction peaks shrink in elastic pp and K^+p scattering, but there is no shrinkage in $\pi^\pm p$ and K^-p scattering, and for $\bar{p}p$ scattering the diffraction cone even broadens with increasing energy [1,2].

The energy dependence of the slope of the diffraction curve is usually described quantitatively by a parameter α' taken from the representation of the differential cross section for elastic scattering in accordance with the single-pole model of the Regge theory

$$\frac{d\sigma(s, t)}{dt} = \frac{d\sigma(s, 0)}{dt} F(t) \exp(2\alpha' t \ln s) \qquad (1)$$

Process	α'_{exp} from Eq. (1), parentheses - momentum range[1)]	$\bar{k}_i$, mb^{-1}	α'_{calc} from Eq. (4), parentheses-momentum range
1	2	3	4
$pp \to pp$	0.685±0.051 [3] (7-25)	0.169± ±0.004	0.5 (10-25)
	0.378±0.193 [3] (15-25)		
$\bar{p}p \to \bar{p}p$	-0,914±0,376 [3] (7-16)	0,173± ±0.003	-0.8 (7-16)
$\pi^+p \to \pi^+p$	0,103±0.074 [1] (7-17)	0.232± ±0.005	0.2 (7-17)
$\pi^-p \to \pi^-p$	0.081 ± 0.073 [1] (7-19)	0.247± ±0.012	0.1 (7-19)
$K^+p \to K^+p$	0.50 ±0.16 [2] (7-15)	0.193± ±0.009	0.5 (7-15)
$K^-p \to K^-p$	-0.172 ± 0.417 [3] (7-16)	0.305± ±0.004	-0.2 (7-16)

The corresponding values of α' are given in the second column of the table. The table lists also the interval of the incident-particle momenta (in the laboratory frame) for which the value of α' was determined.

The purpose of the present note is to show that in spite of the lack of similarity in the behavior of the diffraction-curve slopes in the foregoing elastic scattering processes in the energy region indicated, this behavior can be described, within the limits of experimental accuracy, in a unified fashion. This description starts from the notion that the diffraction cone shrinks with increasing energy for all processes, and from an account of the effect of the total interaction cross sections σ_t on the shape of the diffraction curve.

For small mementum transfer $t \ll 1$ GeV/c^2, the differential elastic-scattering cross sections can be approximated with good accuracy by the exponential [1]

$$\frac{d\sigma_i(s,\ t)}{dt} = \exp[a_i(s) + b_i(s)\cdot t] \tag{2}$$

where the index i denotes any of the scattering processes listed above (i = pp, $\bar{p}p$, K^+p, K^-p, π^+p, π^-p).

It turns out that the energy dependence of the parameter b_i, which determines the slope of the diffraction curve, can be expressed by the formula

$$b_i(s) = 2\beta'[\ln s + R_i\sigma_t(s)_i] \tag{3}$$

where R_i is a constant ($R_i > 0$).

The term $2\beta'\ln s$, the form of which is borrowed from the Regge-pole theory, describes the universal shrinkage of the diffraction curves and the term $2\beta' R_i\sigma_t(s)_i$ describes the influence of the values of $\sigma_t(s)_i$ on the slope of these curves.

Since s is a dimensional quantity, the formula (3) should contain, generally speaking, another term of the form $-2\beta'\ln s_0$. But since we consider only the energy dependence of the slope of the diffraction curve, this term, being independent of s, can be omitted.

From (1) - (3) follows a connection between the parameters α' and β':

$$\alpha' = \beta'\left\{1 + \frac{R_i[\sigma_t(s_1)_i - \sigma_t(s_2)_i]}{\ln s_1 - \ln s_2}\right\} \tag{4}$$

We see from (4) that if the total cross section does not depend on the energy, then $\alpha' = \beta'$, i.e., in this case the diffraction cones should experience for all the scattering processes a similar shrinkage, defined by the parameter β'. On the other hand, if in some energy interval the value of the total cross section decreases, i.e., $\sigma_t(s_1)_i > \sigma_t(s_2)_i$ with $s_1 < s_2$, the shrinkage becomes smaller and may even turn into broadening.

In the energy interval under consideration, the total cross section for the interaction is most constant in the K^+p scattering process. It is therefore natural to choose β' equal to the shrinkage parameter for K^+p scattering, i.e., to put $\beta' = 0.5$. This choice agrees with the data on pp scattering at energies above 10 GeV, where the total cross section of the pp interaction also becomes approximately constant.

We calculated the values of k_i by means of (3) for each value of $b_i(s)$, determined experimentally for n different energies s in [1,2], and for the corresponding values of $\sigma_t(s)_i$. These values turned out to be independent of the energy. The mean values $\overline{k}_i = \Sigma k_i/n$ and the errors $\overline{\Delta k_i} = [\Sigma(\overline{k}_i - k_i)]/n$ are listed in the third column of the table. $\overline{\Delta k_i}$ correspond to the uncertainty in the values of $b_i(s)$. The values of α' calculated from (4) are listed in the fourth column of the table [2]. As can be seen from the table, the calculated values of α' are illustrative not only of the tendency of the diffraction peaks in different scattering processes (shrinkage or broadening), but are also in sufficiently good quantitative agreement with the experimental data. It must be noted here that the choice of β' can be optimized in order to obtain the best agreement with the entire aggregate of data on the slopes of the diffraction curves (the large experimental error in the determination of the parameter α' for K^+p scattering allows us to vary β' over a sufficiently broad range). However, inasmuch as the choice of β' made here enables us to show the applicability of formula (3), no such optimization was made here.

We see from the table that k_{pp} and $k_{\bar{p}p}$ are practically identical. This means that one constant in (3) not only suffices for a description of the behavior of the slope of the diffraction curve of pp and $\bar{p}p$ scattering as a function of the energy, but also explains the difference between these curves at a given energy (the larger $\bar{p}p$-interaction cross section corresponds precisely to a narrower diffraction peak). A single value of k_i can also be used to describe π^-p and π^+p scattering. However, the values of k_i for K^+p and K^-p scattering are essentially different.

We see from (4) that with increasing energy the difference in the behavior of the diffraction curves for different scattering processes becomes smaller and smaller, and that universal shrinkage of the diffraction cones should be observed at sufficiently large energies. Knowing the energy dependence of the total interaction cross sections, we can estimate, assuming constant β', the energies at which the behavior of the diffraction peaks becomes asymptotic with a prescribed degree of accuracy.

The possibility of describing the experimental data by means of a relation similar to (4) demonstrates the correctness of the predictions of the pole model of the theory of complex angular momenta concerning the asymptotic behavior of the diffraction cones.

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[2] K. J. Foley, S. J. Lindenbaum et al., Phys. Rev. Lett. 11, 503 (1963).
[3] S. J. Lindenbaum, Twelfth International Conference on High-energy Physics, Dubna, 1964.
[4] T. F. Kycia, Twelfth International Conference on High-energy Physics, Dubna, 1964.

1) For a larger interval of t it is necessary to take into account the deviation from exponential form and the approximation takes the form $d\sigma_i/dt = \exp(a_i + b_i t + c_i t^2)$, where $c_i = 0.4 b_i$ [1,2].

[2] The vaues of $\sigma_t(s)_i$ used for the corresponding energies were the same as in [1,2]. In addition, we used the values of $(\sigma_t)_{K^-p}$ for 12 and 16 GeV/c and $(\sigma_t)_{\bar{p}p}$ for 16 GeV/c from [4].

RADIATION OF A SYSTEM OF EXCITED NUCLEI IN A CRYSTAL

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1. It is usually assumed that a system of excited particles with radiation wavelength $\lambda < a$ (a - characeristic distance between particles) is equivalent to a system of non-coherent radiators. In a regular crystal, however, there can exist in principle (even when $\lambda \ll a$) excited states whose decay rate is many times larger or smaller than the rate of decay of the non-coherent system.

Let us consider a crystal consisting of N identical nuclei with a low-lying isomer level, and let one of the nuclei be excited. We express the Ψ function of such a state in the form $\Psi = \Sigma_m c_m \varphi_m$, where φ_m describes the state when the m-th nucleus is excited and the others are in the normal state. Because of the identity of the nuclei, the state under consideration can be specified in a large number of ways. If the position of the excited nucleus is strictly defined, i.e., $c_m = \delta_{m0}$, then the probability W_0 of emission of a γ quantum per unit time will be determined by the usual expression for the individual nucleus. On the other hand, we can specify a delocalized state, for example in the form $c_m = N^{-1/2}\exp(i\underset{\sim}{q}\cdot\underset{\sim}{r}_m)$. In this case the lifetime of the excited state will depend on the value of $\underset{\sim}{q}$. Calculating the probability of emission of the γ quantum, we obtain:

$$W = (2\pi/\hbar)\int |M|^2 \left\{ \frac{1}{N} \left| \sum_m \exp[i(\underset{\sim}{k} - \underset{\sim}{q})\cdot\underset{\sim}{r}_m] \right|^2 \right\} \delta(E_0 - E_k)\frac{d^3k}{(2\pi)^3} \qquad (1)$$

Here M is the matrix element corresponding to the transition from the excited to the ground state with emission of a γ quantum.

If q or $|\underset{\sim}{q} + 2\pi\underset{\sim}{b}| \neq k_0/\hbar c$ ($\underset{\sim}{b}$ - reciprocal lattice vector), then $W \sim W_0/N$ in a non-vibrating lattice, and thus the corresponding width Γ_1 decreases macroscopically. Let q or $|\underset{\sim}{q} + 2\pi\underset{\sim}{b}| \approx k_0$. We confine ourselves to an examination of crystals for which the following inequalities are satisfied:

$$(2\pi\hbar c/aN^{1/3}) \gg \Gamma,\ \hbar/t; \qquad a^{-2}\sigma_t N^{1/3} < 1 \qquad (2)$$

If the first inequality is satisfied, the expression in the curly brackets is more smeared out in momentum space, compared with the real energy smearing of the δ function, which is determined by the level width Γ or by the observation time t. The second inequality, being as a rule more stringent, implies that the linear dimension of the crystal is smaller than the absorption length.

Taking (2) into account, we obtain from (1):