[2] The vaues of $\sigma_t(s)_i$ used for the corresponding energies were the same as in [1,2]. In addition, we used the values of $(\sigma_t)_{K^-p}$ for 12 and 16 GeV/c and $(\sigma_t)_{\bar{p}p}$ for 16 GeV/c from [4].

RADIATION OF A SYSTEM OF EXCITED NUCLEI IN A CRYSTAL

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1. It is usually assumed that a system of excited particles with radiation wavelength $\lambda < a$ (a - characeristic distance between particles) is equivalent to a system of non-coherent radiators. In a regular crystal, however, there can exist in principle (even when $\lambda \ll a$) excited states whose decay rate is many times larger or smaller than the rate of decay of the non-coherent system.

Let us consider a crystal consisting of N identical nuclei with a low-lying isomer level, and let one of the nuclei be excited. We express the Ψ function of such a state in the form $\Psi = \Sigma_m c_m \varphi_m$, where φ_m describes the state when the m-th nucleus is excited and the others are in the normal state. Because of the identity of the nuclei, the state under consideration can be specified in a large number of ways. If the position of the excited nucleus is strictly defined, i.e., $c_m = \delta_{m0}$, then the probability W_0 of emission of a γ quantum per unit time will be determined by the usual expression for the individual nucleus. On the other hand, we can specify a delocalized state, for example in the form $c_m = N^{-1/2}\exp(i\underset{\sim}{q}\cdot\underset{\sim}{r}_m)$. In this case the lifetime of the excited state will depend on the value of $\underset{\sim}{q}$. Calculating the probability of emission of the γ quantum, we obtain:

$$W = (2\pi/\hbar)\int |M|^2 \left\{ \frac{1}{N} \left| \sum_m \exp[i(\underset{\sim}{k} - \underset{\sim}{q})\cdot\underset{\sim}{r}_m] \right|^2 \right\} \delta(E_0 - E_k)\frac{d^3k}{(2\pi)^3} \qquad (1)$$

Here M is the matrix element corresponding to the transition from the excited to the ground state with emission of a γ quantum.

If q or $|\underset{\sim}{q} + 2\pi\underset{\sim}{b}| \neq k_0/\hbar c$ ($\underset{\sim}{b}$ - reciprocal lattice vector), then $W \sim W_0/N$ in a non-vibrating lattice, and thus the corresponding width Γ_1 decreases macroscopically. Let q or $|\underset{\sim}{q} + 2\pi\underset{\sim}{b}| \approx k_0$. We confine ourselves to an examination of crystals for which the following inequalities are satisfied:

$$(2\pi\hbar c/aN^{1/3}) \gg \Gamma,\ \hbar/t; \qquad a^{-2}\sigma_t N^{1/3} < 1 \qquad (2)$$

If the first inequality is satisfied, the expression in the curly brackets is more smeared out in momentum space, compared with the real energy smearing of the δ function, which is determined by the level width Γ or by the observation time t. The second inequality, being as a rule more stringent, implies that the linear dimension of the crystal is smaller than the absorption length.

Taking (2) into account, we obtain from (1):

$$W \approx W_0 \pi N^{1/3}/(ka)^2 \tag{3}$$

Thus, there is a strong increase in the probability of γ decay, accompanied by strong peaking of the radiation along the vector $\underset{\sim}{q}$. Both the direction of this vector and the quantity k_0 can be arbitrary here. (The width corresponding to conversion remains unchanged.) These effects can apparently be observed, in principle, by pulsed excitation of the isomer levels of the crystal nuclei (using the Mossbauer transition), i.e., by producing directly a state with $q \approx k_0$ and then observing the intensity of the γ radiation in the direction of the incident beam and at an angle to it, or else by observing the effective change of the conversion coefficient. (We note that the case of a strongly converted level is preferable.)

2. This raises the question whether a state with anomalously large radiation rate appears while the nuclei decay in the crystal ($\lambda \lesssim a$), if at the initial instant all the nuclei are excited (i.e., a state which is in some sense analogous to the "superradiating" state produced in a system with $\lambda > L$ [1], where L is the dimension of the system). The usual expression for the Hamiltonian that describes a system of two-level particles and their interaction with the electromagnetic field, neglecting conversion and vibration of the nuclei, is

$$\hat{H} = H_0 + H^1, \qquad H_0 = (1/2)E_0 \sum_m (\hat{\sigma}_m^z + 1)$$

$$H^1 = \frac{1}{2\sqrt{N}} \sum_m \sum_{k,q} \left\{ M \hat{\sigma}_q^- \hat{Q}_k^+ \exp[i(\underset{\sim}{q} - \underset{\sim}{k})\cdot \underset{\sim}{r}_m] + \text{c.c.} \right\} \tag{4}$$

$$\hat{\sigma}_q^{\pm} = N^{-1/2} \sum_m \hat{\sigma}_m^{\pm} \exp(\mp i \underset{\sim}{q}\cdot \underset{\sim}{r}_m)$$

Here $\hat{\sigma}^z$ and $\hat{\sigma}^{\pm}$ are the usual Pauli matrices. If we confine ourselves to that stage of the decay when the number of nuclei in normal state, βN, is still small compared with N, then the operators $\hat{\sigma}_q^{\pm}$ can be replaced, in complete analogy with the theory of spin waves [2], by second-quantization operators with Bose commutation rules: $\hat{\sigma}_q^+ = 2\hat{b}_q$ and $\hat{\sigma}_q^- = 2\hat{b}_q^+$. The creation of each γ quantum is accompanied by the appearance of one normal state which has a non-localized character and is actually a collective excitation (with negative energy) in a system of almost completely excited nuclei. These Bose-type excitations will be called de-excitons. It is clear that the state of the system is completely specified by the set n_q of the de-excitons occupation numbers, with the sum $\Sigma_q n_q$ equal to the number of emitted γ quanta (we neglect here the dispersion of the de-excitons).

We define in accordance with [4] the number of decays per unit time, for a state characterized by the set $\{n_q\}$, by means of the expression

$$W(\{n_q\}) = (2\pi/\hbar)\frac{1}{N} \sum_{k,q} |M|^2 \Big| \sum_m \exp[i(\underset{\sim}{q} - \underset{\sim}{k})\cdot \underset{\sim}{r}_m] \Big|^2 (n_q + 1)\delta(E_0 - E_k) \tag{5}$$

At the initial instant $n_q = 0$ and we arrive at the obvious result $W = NW_0$. Later on, however, the picture changes radically. According to (5), spontaneous emission of photons is stimulated emission with respect to the de-excitons, and the increase in the rate will depend on the

values of the numbers n_q. In accordance with (5), q can differ from k_0 only within the limits $2\pi/aN^{1/3}$. Consequently, de-excitons will be created in phase spece only in a narrow spherical layer near $q = k_0$, the number of states being

$$\sim 4\pi k_0^2(2\pi)^{-3}(Na^3)(2\pi/aN^{1/3}) = (k_0a)^2N^{2/3}/\pi$$

Taking this result into account, we have for the state arising after βN decays

$$W = W_0[\pi\beta N^{4/3}/(k_0a)^2] \tag{6}$$

Thus, the rate of decay increases macroscopically; in an ideal crystal this should hold for perfectly arbitrary values of k_0. It is also easy to find the time variation of the radiation intensity by solving the equation

$$\dot{n}_q = W_q(n_q + 1) - n_q/\tau_q; \quad W_q \approx NW_0\pi/(k_0a)^2N^{2/3} = gW_0N^{1/3}$$

(We have introduced here, purely formally, the relaxation term for the de-excitons; we have hitherto assumed tacitly that $1/\tau_q \ll W_q$.) It follows therefore that when $W_q > 1/\tau_q$ the radiation intensity increases exponentially. We note that anisotropy of W_q causes immediately the radiation to become sharply anisotropic.

3. In a very recent communication [3] it is stated without proof that a "superradiating" state can be produced in a crystal only if the lattice and the nucleus are artificially chosen in such a way that $k_0 = 2\pi b$. We think that this conclusion is incorrect, because, as seen from the foregoing, the effect can exist for arbitrary k_0. Furthermore, the two other statements, that the radiation intensity is proportional to N^2 and that the radiation itself is directed along the vector $\underset{\sim}{b}$, are likewise incorrect.

Similar statements concerning the need for satisfying the condition $k_0 = 2\pi b$ to increase the rate of decay of the nucleus and concerning the macroscopic peaking of the radiation along the vector $\underset{\sim}{b}$, are contained also in [4]. Unfortunately, the main result, whereby the lifetime in the decay of a spatially fixed excited nucleus decreases macroscopically if this condition is satisfied, is likewise in error.

4. We note that the considerations presented above have an idealized character and the analysis of the concrete cases calls for inclusion of many additional factors.

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